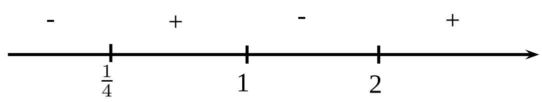
JC Revision: Inequalities Solutions

1. (a)

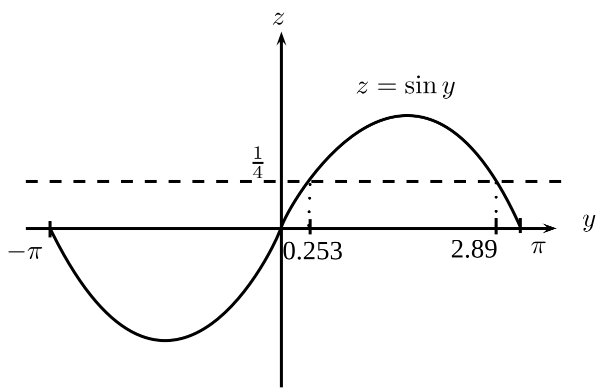
$$\frac{3}{1-x} \leq 5-4x, \quad x \neq 1$$
$$\frac{3}{1-x} - 5 + 4x \leq 0$$
$$\frac{3}{1-x} + \frac{(-5+4x)(1-x)}{1-x} \leq 0$$
$$\frac{3+(-5+5x+4x-4x^2)}{1-x} \leq 0$$
$$\frac{-2+9x-4x^2}{1-x} \leq 0$$
$$\frac{(-4x+1)(x-2)}{1-x} \leq 0$$



Since $x \neq 1$, $x \leq \frac{1}{4}, 1 < x \leq 2$

(b)
From $\frac{3}{1-x} \leq 5-4x$, replacing x by $\sin y$,
$\frac{3}{1-\sin y} \leq 5 - 4\sin y$

$\therefore \sin y \leq \frac{1}{4}, \quad 1 < \sin y \leq 2$ (N.A)



From the graph,
$-\pi \leq y \leq 0.253, \quad 2.89 \leq y \leq \pi$

2.

$$\frac{4x^2+4x+1}{x^2+x+1} > 0$$
$$\frac{(2x+1)^2}{x^2+x+1} > 0$$
$$\frac{(2x+1)^2}{(x+\frac{1}{2})^2 - (\frac{1}{2})^2 + 1} > 0$$
$$\frac{(2x+1)^2}{(x+\frac{1}{2})^2 + \frac{3}{4}} > 0$$

Since $(x+\frac{1}{2})^2 + \frac{3}{4}$ is always positive, we get,

$$(2x+1)^2 > 0$$

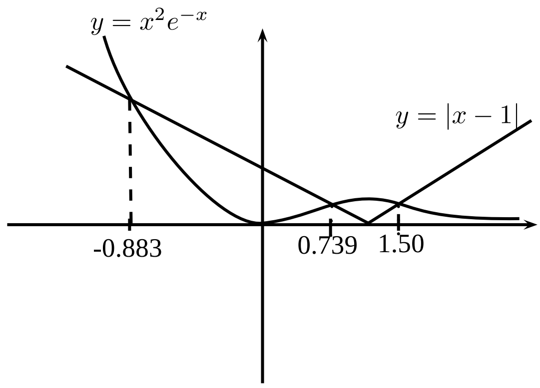
$\therefore x \in \mathbb{R}, x \neq -\frac{1}{2}$
(since the only value that makes this inequality false is when $x = -\frac{1}{2}$.)

From $\frac{4x^2+4x+1}{x^2+x+1} > 0$, replacing x by $\frac{1}{x}$,

$$\frac{\frac{4}{x^2}+\frac{4}{x}+1}{\frac{1}{x^2}+\frac{1}{x}+1} > 0$$
$$\frac{\frac{4}{x^2}+\frac{4x}{x^2}+\frac{x^2}{x^2}}{\frac{1}{x^2}+\frac{x}{x^2}+\frac{x^2}{x^2}} > 0$$
$$\frac{4+4x+x^2}{1+x+x^2} > 0$$

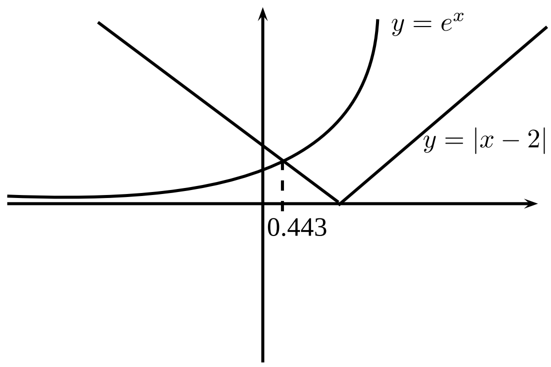
Replacing x by $\frac{1}{x}$ for the answer in the first part,
$\frac{1}{x} \in \mathbb{R}, \frac{1}{x} \neq \frac{-1}{2}$
$\therefore x \in \mathbb{R}, x \neq -2$

3. We want $x^2e^{-x} < |x-1|$. Using GC,



$x > 1.50$ or $-0.883 < x < 0.739$.

4.



$x > 0.443$.

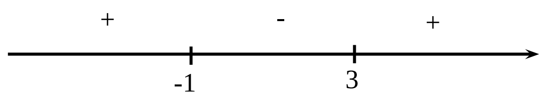
From $|x-2| < e^x$, replace x by $x+1$ to obtain:

$$|x+1-2| < e^{x+1}$$
$$\frac{|x-1|}{e} < e^x$$

$$\therefore x+1 > 0.443$$
$$x > -0.557$$

5.

$$x \neq 3$$
$$\frac{4x}{x-3} - 1 \geq 0$$
$$\frac{4x}{x-3} - \frac{x-3}{x-3} \geq 0$$
$$\frac{4x-x+3}{x-3} \geq 0$$
$$\frac{3x+3}{x-3} \geq 0$$
$$\frac{x+1}{x-3} \geq 0$$



$\therefore x \leq -1$ or $x \geq 3$
Since $x \neq 3$, $x \leq -1$ or $x > 3$

From $\frac{4x}{x-3} \geq 1$, replace x by $|x|$,

$$\frac{4|x|}{|x|-3} \geq 1$$

$|x| \leq -1$ (N.A.) or $|x| > 3$
$\implies x < -3$ or $x > 3$