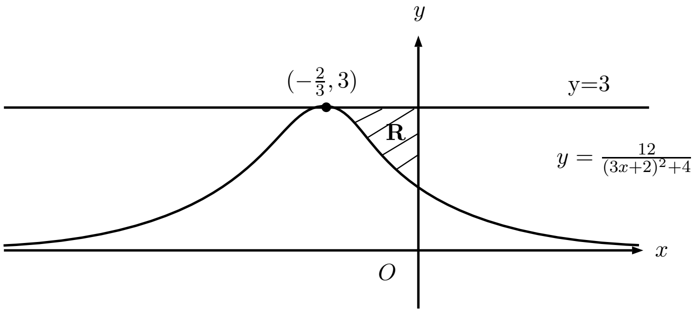
Integration Application Revision

1. [2016/JJC/Prelim/I/3]

The diagram shows the curve C with equation $y = \frac{12}{(3x+2)^2+4}$ which has a turning point at $(-\frac{2}{3}, 3)$. The region bound by C, the y-axis and the line $y = 3$.



i. Find the exact area of R. [5]

ii. R is rotated through 2π radians about the y-axis. Find the volume of the solid of revolution formed, giving your answer to 4 decimal places. [3]

[(i) $2 - \frac{\pi}{2}$ (ii) 0.5125]

2. [2016/NJC/Prelim/I/10]

(a) Using partial fractions, find $\displaystyle\int \frac{5x^2 - 2x + 7}{(1-x)(2x^2+3)}\, \mathrm{d}x$. [6]

(b) i. Differentiate $\sin(e^{-x})$ with respect to x. [1]

ii. Obtain a formula for $\displaystyle\int_0^n e^{-2x} \cos(e^{-x})\, \mathrm{d}x$ in terms of n, where $n > 0$. [3]

iii. Hence find $\displaystyle\int_0^\infty e^{-2x} \cos(e^{-x})\, \mathrm{d}x$ exactly. [2]

[(a) $-2\ln|1-x| - \frac{1}{4}\ln(2x^2+3) + \frac{1}{\sqrt{6}}\tan^{-1}\left(\sqrt{\frac{2}{3}}x\right) + c$ (b)i) $-e^{-x}\cos(e^{-x})$ (ii) $-e^{-n}\sin(e^{-n}) - \cos(e^{-n}) + \sin 1 + \cos 1$ (iii) $\sin 1 + \cos 1 - 1$]

3. [2016/PJC/Prelim/I/9]

(a) i. If $t = \tan\frac{\theta}{2}$, show that $\sin\theta = \frac{2t}{1+t^2}$. [2]

ii. Use the substitution $t = \tan\frac{\theta}{2}$ to find the exact value of $\displaystyle\int_0^{\frac{\pi}{2}} \left(\frac{\tan\frac{\theta}{2} + 1}{\sin\theta + 1}\right) \mathrm{d}\theta$. [5]

(b) Find $\displaystyle\int e^{2v}\cos 3v\, \mathrm{d}v$. [4]

[(a)(ii) $2\ln 2$ (b) $\frac{3}{13}e^{2v}\sin 3v + \frac{2}{13}e^{2v}\cos(3v) + c$]

4. [2016/SRJC/Prelim/II/1]

(a) If $0 < a < b$, solve $\displaystyle\int_0^b x|a-x|\, \mathrm{d}x$, leaving your answers in terms of a and b. [2]

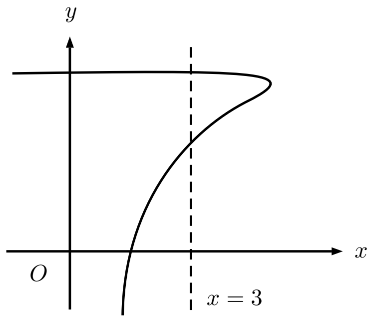
(b) i. Find $\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{3-x}{\sqrt{1-x}}\right)$. [1]

ii. Find $\displaystyle\int \frac{3-x}{x^2-3x+2}\, \mathrm{d}x$. [2]

iii. Hence find $\displaystyle\int \frac{1+x}{(1-x)^{\frac{3}{2}}} \tan^{-1}\sqrt{1-x}\, \mathrm{d}x$. [3]

[(a) $\frac{a^3}{3}+\frac{b^3}{3}-\frac{ab^2}{2}$ (b)i) $\frac{x+1}{2(1-x)^{\frac{3}{2}}}$ ii) $\ln|x-2|-2\ln|x-1|+c$ (iii) $2\left(\frac{3-x}{\sqrt{1-x}}\right)\tan^{-1}\sqrt{1-x}+\ln|x-2|-2\ln|x-1|+c$]

5. [2016/TJC/Prelim/I/10]



The diagram shows the curve with parametric equations

$$x=2t+t^2, y=\frac{1}{(1-t)^2}, \text{ for } t<1.$$

The curve has a vertical asymptote $x=3$.

i. Find the coordinates of the points where the curve cuts the y-axis. [2]

ii. Find the equation of the tangent to the curve that is parallel to the y-axis. [4]

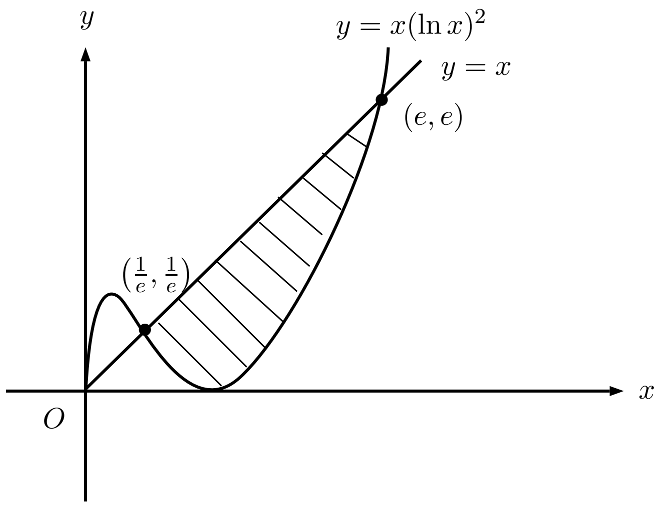
iii. Express the area of the finite region bounded by the curve and the y-axis in the form $\int_a^b f(t)\ \mathrm{d}t$, where a, b and f are to be determined. Use the substitution $u=1-t$ to find this area, leaving your answer in exact form. [5]

[(i) $(0,1)$ and $(0,\frac{1}{9})$ (ii) $x=-1$ (iii) $\frac{8}{3}-2\ln 3$]

6. [2016/TJC/Prelim/II/4]

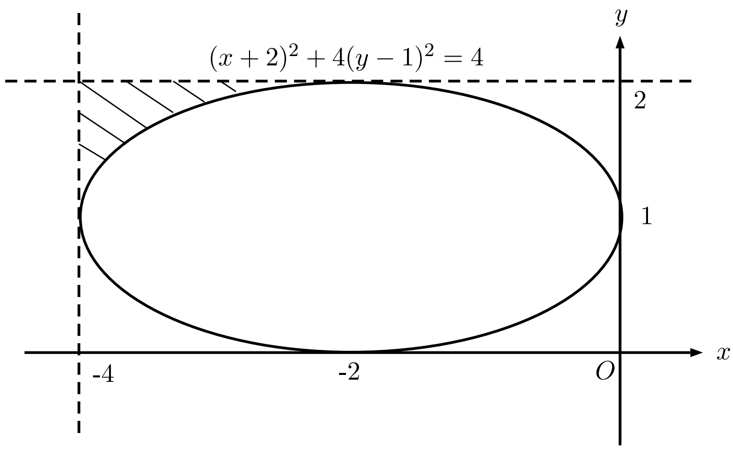
(a) The diagram below shows the graphs of $y=x(\ln x)^2$ and $y=x$. The two graphs intersect at the points $(\frac{1}{e},\frac{1}{e})$ and (e,e).

Find the exact area of the shaded region bounded by the graphs of $y=x(\ln x)^2$ and $y=x$. [5]



(b) Hence, without integrating, find the exact area of the region bounded by the graphs of $y=x(\ln x)^2$ and the lines $y=e$ and $x=\frac{1}{e}$. [2]

(c) Find the volume of the solid formed when the shaded region bounded by the lines $x = -4, y = 2$ and the ellipse $(x+2)^2 + 4(y-1)^2 = 4$ is rotated through 2π radians about the y-axis. Give your answer correct to 1 decimal place. [4]



[(a) $\frac{1}{4}\left(e^2 + \frac{3}{e^2}\right)$ (b) $\frac{1}{4}\left(e^2 + \frac{3}{e^2}\right) + \frac{1}{2}\left(e - \frac{1}{e}\right)^2$ (b) 9.6 units^3 (c)]