Differentiation J1 Revision

1. [2011/YJC/Prelim/I/8b]

   The equation of a curve C is $x^3 + 2y^3 + 3xy = k$, where k is a constant.

   i. Find $\frac{dy}{dx}$ in terms of x and y. [3]

   ii. It is given that C has a tangent which is parallel to the x-axis. Show that the x-coordinate of the point of contact of the tangent with C must satisfy

   $$2x^6 + 2x^3 + k = 0.$$

   [3]

   iii. Hence, find the values of k when the line $y = -1$ is a tangent to the curve C. [2]

   [(i) $\frac{dy}{dx} = \frac{-y-x^2}{2y^2+x}$ (iii) $k = 0, k = -4$]

2. [2011/SRJC/Prelim/II/3]

   A curve C is defined by the parametric equations

   $$x = \theta - \cos\theta,\ y = \theta + \cos\theta,\ 0 \leq \theta \leq 2\pi$$

   (a) Find the equation of the tangent to the curve C that is parallel to the y-axis. [3]

   (b) Find the exact equation of the tangent to the curve at the point P where $\theta = \frac{\pi}{6}$. [3]

   (c) Hence, determine whether the tangent at point P will meet the curve C again. [3]

   [(a) $x = \frac{3\pi}{2}$ (b) $y = \frac{1}{3}x + \frac{\pi}{9} + \frac{2}{\sqrt{3}}$ (c) yes]

3. [2011/SAJC/Prelim/I/12]

   (a) A curve has parametric equations

   $$x = \theta - \sin\theta,\ y = 1 - \cos\theta,\ \text{for } 0 < \theta < 2\pi.$$

   Find the equation of the tangent parallel to the x-axis. [3]

   (b) The normal to the curve at the point with parameter $\frac{2\pi}{3}$, meets the x- and y-axes at P and Q respectively.

   Show that the equation of the normal is $y = -\sqrt{3}x + \frac{2\sqrt{3}\pi}{3}$.

   Hence find the area of the triangle OPQ. [5]

   (c) Given that θ is increasing at a rate of 2 radians per second, find the rate of change for $\frac{dy}{dx}$ of the curve at $\theta = \frac{\pi}{3}$. [3]

   [Hint: rate of change of $\frac{dy}{dx} = \frac{d\left(\frac{dy}{dx}\right)}{dt}$]

   [(a) $y = 2$ (b) $\frac{2\sqrt{3}\pi^2}{9}$ units^2 (c) -4 units/s]

4. [2011/RI/Prelim/II/3]

   A curve C has parametric equations

   $$x = a\sin^4\theta,\ y = b\cos^4\theta,$$

   where a and b are positive constants and $0 < \theta < \frac{\pi}{2}$.

   i. Sketch the curve C in the case where $a = 2$ and $b = 1$. [1]

   The tangent to C at the point $(a\sin^4\theta, b\cos^4\theta)$ cuts the x-axis at the point P and the y-axis at the point Q.

ii. Show that, at the point $(a\sin^4\theta, b\cos^4\theta)$ on C, $\frac{dy}{dx} = -\frac{b}{a}\cot^2\theta$. [3]

iii. Show that the coordinates of P are $(a\sin^2\theta,\ 0)$ and find the coordinates of Q, simplifying your answer. [3]

iv. The area of triangle OPQ is denoted by A, where O refers to the origin. The value of A varies as θ varies between 0 and $\frac{\pi}{2}$. Determine the exact value of θ at which A attains a maximum, and write down the corresponding value of A. [3]

(It is not necessary to verify the nature of the stationary value of A.)

[(iii) $(0, b\cos^2\theta)$ (iv) A is maximum when $\theta = \frac{\pi}{4}$, $A = \frac{ab}{8}$]

5. [2009/CJC/Prelim/I/6]

Given that $y = e^{1+x^2}$, prove that
$$\frac{d^2y}{dx^2} = 2\left(y + x\frac{dy}{dx}\right).$$

6. [2010/SAJC/Prelim/I/9]

It is given $\frac{dy}{dx} = \frac{e^{\tan^{-1}x}}{1+x^2}$, where $\tan^{-1}x$ denotes the principal value.

Show that $(1+x^2)\frac{d^2y}{dx^2} + (2x-1)\frac{dy}{dx} = 0$.

7. [2012/TJC/Prelim/I/6]

Given that $y = \ln(1+\sin x)$, show that

i. $e^y\frac{dy}{dx} = \cos x$, [2]

ii. $\frac{d^3y}{dx^3} + 3\left(\frac{d^2y}{dx^2}\right)\left(\frac{dy}{dx}\right) + \left(\frac{dy}{dx}\right)^3 + \frac{dy}{dx} = 0$. [3]

8. [2010/ACJC/Prelim/I/13]

The curve has the parametric equations
$$x = \frac{5}{1+t^2},\ y = \tan^{-1}t.$$

(a) Sketch the curve for $-2 \le t \le 2$. [1]

(b) Find the cartesian equations of the tangent and normal to the curve at the point where $t = 1$. [5]

(c) Find the area enclosed by the x-axis, the tangent and the normal at the point where $t = 1$. [3]

[(b) equation of tangent: $y = -\frac{1}{5}x + (\frac{1}{2} + \frac{\pi}{4})$ (c) $\frac{13\pi^2}{80}$]