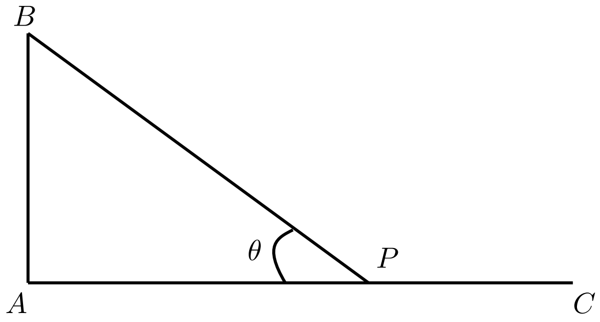
1. [2016/TJC/Prelim/I/5]




In the diagram, $A$ and $C$ are fixed points 500 m apart on horizontal ground. Initially, a drone is at point $A$ and an observer is standing at point $C$. The drone starts to ascend vertically at a steady rate of 3 m $s^{-1}$ as the observer starts to walk towards $A$ with a steady speed of 4 m $s^{-1}$. At time $t$, the drone is at point $B$ and the observer is at point $P$.

Given that the angle $APB$ is $\theta$ radians, show that $\theta = \tan^{-1}\left(\frac{3t}{500-4t}\right)$. [2]

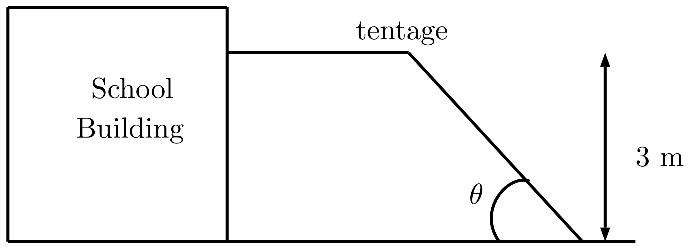
   i. Find $\frac{d\theta}{dt}$ in terms of $t$. [2]

   ii. Using differentiation, find the time $t$ when the rate of change of $\theta$ is maximum. [4]

[(i) $\frac{d\theta}{dt} = \frac{60}{t^2-160t+10000}$ (ii) $t = 80$]

2. [2015/HCI/Prelim/I/4]

A group of student councillors bought an 8 m long piece of tarpaulin canvas to build a tentage for a school event. The canvas would extend diagonally at an angle of $\theta$ from the ground to a height of 3 m, where it will then stretch horizontally to the school building (see diagram for the cross-sectional view).




   i. Given that the total cross-sectional area covered by the canvas is $A$ m $^2$, show that $A = 24 - 9\operatorname{cosec}\theta + \frac{9}{2}\cot\theta$. [2]

   ii. Find, by differentiation, the largest possible value of $A$. [5]

[(ii) $16.2\text{m}^2$]

3. [2015/MJC/Prelim/I/11]

A curve has parametric equations

$$x = 2\cos t,\ y = \sin t,\ \text{for } 0 \le t \le 2\pi.$$

Show that the equations of the tangent and normal to $C$ at the point $P$ with parameter $\theta$ are $(\cos\theta)x + (2\sin\theta)y = 2$ and $(2\sin\theta)x - (\cos\theta)y = 3\sin\theta\cos\theta$ respectively. [5]

   i. Show algebraically that the tangent to $C$ at the point $P$ does not cut the curve $C$ again. [3]

   ii. The normal to $C$ at the point $P$ cuts the $x$-axis and $y$-axis at points $A$ and $B$ respectively. By finding the mid-point of $AB$, determine a cartesian equation of the locus of the mid-point of $AB$ as $\theta$ varies. Hence describe the locus formed. [6]

[(ii) $\left(\frac{3}{4}\cos\theta, -\frac{3}{2}\sin\theta\right)$]

4. [2015/VJC/Prelim/I/9]

A curve $C$ has parametric equations

$$x = e^{\theta}\sin\theta, y = e^{\theta}\cos\theta$$

where $-\frac{\pi}{2} < \theta \leq \frac{\pi}{2}$.

i. Sketch $C$, indicating clearly the axial intercepts. [2]

ii. $C$ cuts the $y$- and $x$-axes at points $A$ and $B$ respectively. A particle moves along $C$ from $A$ to $B$, with its $x$-coordinate increasing at a constant rate of 0.1 units per second. Find the exact rate of change of its $y$-coordinate when $x = \frac{1}{2}e^{\frac{\pi}{6}}$. [3]

iii. The tangent at the point $P$ on $C$ is parallel to the $y$-axis. Find the equation of this tangent. [4]

iv. The point $Q$ on $C$ is such that angle $POQ = \frac{\pi}{2}$. Find the area of triangle $OPQ$. [5]

[(ii) $\frac{\sqrt{3}-1}{10(\sqrt{3}+1)}$ (iii) $x = -\frac{1}{\sqrt{2}}e^{-\frac{\pi}{4}}$ (iv) $\frac{1}{2}$ units$^2$]