Graphing + Maclaurin's J1 Revision

1. [2012/RVHS/Prelim/I/7]

   The curve C has equation $y = \frac{2x^2+13x+23}{x+3}$.

   (a) Prove, using an algebraic method, that C cannot lie between two values which are to be determined. [3]

   (b) State the equations of the asymptotes of C. [2]

   (c) Draw a sketch of C, showing clearly any axial intercepts, asymptotes and stationary points. [3]

   (d) By considering a circle with centre at the points $(-3, 1)$, find the range of values of k such that the equation $(x+3)^2 + (\frac{2x^2+12x+20}{x+3})^2 = k^2$ has a positive root. [3]

   [(b) $y = 2x + 7, x = -3$ (d) $k < -\frac{\sqrt{481}}{3}$ or $k > \frac{\sqrt{481}}{3}$]

2. [2011/TPJC/Prelim/II/1]

   (a) Sketch the graph of $y = \frac{2x+1}{x-4}$, giving the equations of any asymptotes and the coordinates of any points of intersection with the x- and y-axes. [3]

   (b) Hence find the least value of k where k is a positive integer, such that the equation $(\frac{2x+1}{x-4})^2 = kx$ has 3 real roots. [2]

   (b) $k = 1$]

3. [2011/ACJC/II/5]

   The curves C_1 and C_2 have equations $9y^2 = (x+k)^2 - 9$ and $\frac{x^2}{k^2} + y^2 = 1$ respectively, where k is a real constant such that $k > 3$.

   i. On the same diagram, sketch the graphs of C_1 and C_2, stating clearly the coordinates of any points of intersection with the axes and the equations of any asymptotes. [2]

   ii. Find the range of values of the positive constant a such that the equation

   $$\frac{x^2}{a^2} + \frac{(x+k)^2 - 9}{9} = 1$$

   has four real roots. [5]

   [(i) C_1: $(3-k,\ 0)$, $(-3-k,\ 0)$, $(0,\ \pm\sqrt{\frac{k^2}{9} - 1})$; $y = \pm\frac{x+k}{3}$; C_2: $(\pm k, 0)$, $(0,\ \pm 1)$ (ii) $a > k + 3$]

4. [2009/DHS/I/3]

   (a) Sketch the curve $9x^2 - 4y^2 - 36x = 0$, showing clearly the axis-intercepts and the equations of the asymptotes, if any. [3]

   (b) The curve C has the equation $y = \frac{x}{2} + \frac{A}{x-1}$, where A is a constant.

      i. State the equations of the asymptotes of C. [2]

      ii. Find the range of values of A such that C does not have any stationary points. [2]

   [(a) $y = -\frac{3}{2}x + 3$ b(i) $y = \frac{x}{2}, x = 1$ b(ii) $A < 0$]

5. [2009/AJC/I/9]

   The curve C has equation $y = \frac{x^2-ax+b}{x-2}$ where a, b are constants and $b \neq 0$.

   (a) Find $\frac{dy}{dx}$ and hence show that if the curve C has 2 stationary points, then $b > 2a - 4$. [3]

   (b) Given that $-1 < b < 0$, sketch the curve C for $a = 5$, showing clearly all its asymptotes. [2]

   Hence find the largest possible value of k such that the equation $2x^2 - 10x - 1 = (mx - 3)(2x - 4)$ has exactly 2 real roots for all $m < k$. [2]

   [(b) Largest value of k is 1]

6. [2012/TJC/Prelim/I/6]

   (a) Given that $y = \ln(1 + \sin x)$, show that

      i. $e^y \frac{dy}{dx} = \cos x$, [2]

      ii. $\frac{d^3y}{dx^3} + 3\left(\frac{d^2y}{dx^2}\right)\left(\frac{dy}{dx}\right) + \left(\frac{dy}{dx}\right)^3 + \frac{dy}{dx} = 0$. [3]

   (b) Find the Maclaurin's series for y up to and including the term in x^3. [3]

   (c) Hence, or otherwise, show that $\frac{\cos x}{1+\sin x} \approx 1 - x + \frac{x^2}{2}$. [2]

   [(b) $y = x - \frac{1}{2}x^2 + \frac{1}{6}x^3 + \ldots$]

7. [2011/SAJC/Prelim/I/8]

   Given that $y = \ln(1 + 2x)$,

   (a) show that $(1 + 2x)\frac{dy}{dx} = 2$ [1]

   (b) By further differentiation of the result in (a), find the Maclaurin's series for y in ascending powers of x up to and including the term in x^3. [5]

   (c) Deduce that $\ln\left(\sqrt{\frac{1+2x}{1-2x}}\right) \approx px + qx^3$, where p and q are constants to be determined. [1]

   [(b) $y = 2x - 2x^2 + \frac{8}{3}x^3 + \ldots$ (c) $p = 2, q = \frac{8}{3}$]