# Vectors Revision 2

1. [2016/ACJC/Prelim/I/11)]

   Referred to the origin $O$, the position vectors of the points $A, B$ and $C$ are $\mathbf{a}, \mathbf{b}$ and $\mathbf{c}$ respectively. Given that $\mathbf{a} \times \mathbf{b} = 4\mathbf{a} \times \mathbf{c}$, where $\mathbf{b} \neq 4\mathbf{c}$ and $\mathbf{a}$ is a non-zero vector,

   i. show that $\mathbf{b} - 4\mathbf{c} = \alpha\mathbf{a}$ where $\alpha$ is a scalar. [1]

   ii. Hence evaluate $|\mathbf{b} \times \mathbf{c}|$, given that the area of triangle $OAB$ is $\sqrt{126}$ and $\alpha = \sqrt{3}$. [2]

   iii. Give the geometrical meaning of $|\mathbf{b} \times \mathbf{c}|$. [1]

   iv. It is also given that $\mathbf{b}$ is a unit vector, $|\mathbf{a}| = 5, |\mathbf{c}| = 2$ and $\mathbf{b} - 4\mathbf{c} = \sqrt{3}\mathbf{a}$.

   By considering $(\mathbf{b} - 4\mathbf{c}) \cdot (\mathbf{b} - 4\mathbf{c})$, find the angle between $\mathbf{b}$ and $\mathbf{c}$. [3]

   [(ii) $\frac{\sqrt{378}}{2}$ (iv) $128.7°$]

2. [2016/ACJC/Prelim/II/4)]

   The equations of three planes $p_1, p_2, p_3$ are

$$2x + 3y - 6z = 10,$$

$$-2x - 3y + 6z = a,$$

$$x + y + bz = 5,$$

   respectively, where $a, b$ are constants.

   The planes $p_1$ and $p_3$ intersect in the line $l$ with cartesian equation $\frac{5-x}{3} = \frac{y}{4} = z$.

   i. Show that $b = -1$. [2]

   ii. The point $S$ lies on $p_1$ and the point $R$ has coordinates $(-2, 4, 1)$. Given that $RS$ is perpendicular to $p_3$, find the coordinates of $S$. [4]

   The planes $p_1$ and $p_2$ are $\frac{8}{7}$ units apart.

   iii. Given that $a < 0$, find the possible values of $a$. [4]

   iv. The point $P$ with coordinates $(5, 2, c)$ lies on $p_1$. Find the value of $c$. [1]

   v. The point $F$ is the foot of perpendicular from $P$ to the line $l$. The point $Q$ is the reflection of $F$ in the plane $p_2$. Find the distance $PF$ and hence find the area of triangle $FPQ$. [4]

   [(ii) $\begin{pmatrix} -\frac{14}{11} \\ \frac{52}{11} \\ \frac{3}{11} \end{pmatrix}$ (iii) $a = -2$ or $-18$ (iv) $c = 1$ (v) $PF = 1.3728$, Area $PFQ = 1.57$]

3. [2016/IJC/Prelim/I/5)]

   Referred to the origin $O$, the points $A$ and $B$ are such that $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = b$. The point $C$ on $OA$ is such $OC : OA = 1 : 3$. The line $l$ passes through the points $A$ and $B$. It is given that angle $BOA = 60°$ and $|\mathbf{a}| = 3|\mathbf{b}|$.

   i. By considering $(\mathbf{b} - \mathbf{a}) \cdot (\mathbf{b} - \mathbf{a})$, or otherwise, express $|\mathbf{b} - \mathbf{a}|$ in the form $k|\mathbf{b}|$, where $k$ is a constant to be found in exact form. [3]

   ii. Find, in terms of $|\mathbf{b}|$, the shortest distance from $C$ to $l$. [5]

   [(i) $\sqrt{7}|\mathbf{b}|$ (ii) $\sqrt{\frac{3}{7}}|\mathbf{b}|$ ]

4. [2016/NYJC/Prelim/I/4)]

   Referred to the origin, the points $A$ and $B$ have position vectors $\mathbf{a}$ and $\mathbf{b}$ respectively. A point $C$ is such that $OACB$ forms a parallelogram. Given that $M$ is the mid-point of $AC$, find the position vector of point $N$ if $M$ lies on $ON$ produced such that $OM : ON$ is in the ratio $3 : 2$. Hence show that $A, B$ and $N$ are collinear. [4]

Point $P$ is on $AB$ is such that $MP$ is perpendicular to $AB$. Given that angle $AOB$ is $60°$, $|\mathbf{a}| = 2$ and $|\mathbf{b}| = 3$, find the position vector of $P$ in terms of $\mathbf{a}$ and $\mathbf{b}$. [4]

[$\overrightarrow{ON} = \frac{1}{3}(2\mathbf{a} + \mathbf{b})$, $\overrightarrow{OP} = \mathbf{a} + \frac{3}{7}(\mathbf{b} - \mathbf{a}) = \frac{1}{7}(4\mathbf{a} + 3\mathbf{b})$]

5. [2016/PJC/Prelim/I/10)]

The point $A$ has coordinates $(18, 2, 0)$. The plane $p_1$ has the equation $x + 3y + z = a$, where $a$ is a constant. It is given that $p_1$ contains the line $l_1$ with equation $\frac{x-1}{2} = y = \frac{z-1}{-5}$.

i. Show that $a = 2$. [2]

ii. Find the coordinates of the foot of the perpendicular from the point $A$ to $p_1$. [3]

iii. $B$ is given to be a general point on $l_1$. Find an expression for the distance between the point $A$ and $B$. Hence find the position vector of $B$ that is nearest to $A$. [4]

iv. The planes $p_2$ and $p_3$ have the equations $x + z = 1$ and $2x + by + z = 4$ respectively, where $b$ is a constant.

Given that $p_2$ and $p_3$ intersect at $l_2$, show that $l_2$ is parallel to the vector $\begin{pmatrix} -b \\ 1 \\ b \end{pmatrix}$. By finding a point that lies on both planes, find a vector equation of $l_2$. [3]

[(ii) $N(16, -4, -2)$ (iii) $|\overrightarrow{AB}| = \sqrt{294 - 72\lambda + 30\lambda^2}$, $\overrightarrow{OB} = \begin{pmatrix} \frac{17}{5} \\ \frac{6}{5} \\ 7 \end{pmatrix}$ (iv) $l_2 : r = \begin{pmatrix} 3 \\ 0 \\ -2 \end{pmatrix} + \mu \begin{pmatrix} -b \\ 1 \\ b \end{pmatrix}, \mu \in \mathbb{R}$ (The position vector you choose can be different)]

6. [2016/SRJC/Prelim/I/6)]

(a) The equations of two planes $p_1$ and $p_2$ are

$$x + 4y + 2z = 7,$$

$$3x + \lambda y + 4z = \mu,$$

respectively, where $\lambda$ and $\mu$ are constants.

i. Given that the two planes intersect in a line $l$, with a vector equation given by

$$\mathbf{r} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} + s \begin{pmatrix} -2 \\ 1 \\ -1 \end{pmatrix}, s \in \mathbb{R},$$

show that the value of $\lambda$ is 10 and find the value of $\mu$. [3]

ii. If plane $p_3$ is the reflection of $p_1$ in $p_2$, find the acute angle between $p_1$ and $p_3$. [2]

(b) Relative to the origin $O$, the points $A, B, C$ and $D$ have position vectors $\mathbf{a}, \mathbf{b}, \mathbf{c}$ and $\mathbf{d}$ respectively. It is given that $\lambda$ and $\mu$ are non-zero numbers such that $\lambda\mathbf{a} + \mu\mathbf{b} - \mathbf{c} = 0$ and $\lambda + \mu = 1$,

i. Show that $A, B$ and $C$ are collinear. [3]

ii. If $O$ is not on the line $AC$ and $|\mathbf{c} \times \mathbf{a}|(\mathbf{b} - \mathbf{a}) = (\mathbf{c} \cdot \mathbf{d})\mathbf{d}$, determine the relationship between $\overrightarrow{AC}$ and $\overrightarrow{OD}$, explaining your answer clearly. [2]

[a(i) $\mu = 17$ (ii) $11.0°$]