List MF15

LIST OF FORMULAE

AND

STATISTICAL TABLES

for

Mathematics

For use from 2010 in all papers for the H1, H2 and H3 Mathematics syllabuses.

This document consists of **8** printed pages.

PURE MATHEMATICS

Algebraic series

Binomial expansion:

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \binom{n}{3}a^{n-3}b^3 + \ldots + b^n, \text{ where } n \text{ is a positive}$$

integer and $\binom{n}{r} = \dfrac{n!}{r!(n-r)!}$

Maclaurin's expansion:

$$f(x) = f(0) + x\,f'(0) + \frac{x^2}{2!}f''(0) + \ldots + \frac{x^n}{n!}f^{(n)}(0) + \ldots$$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \ldots + \frac{n(n-1)\ldots(n-r+1)}{r!}x^r + \ldots \qquad \left(|x|<1\right)$$

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots + \frac{x^r}{r!} + \ldots \qquad (\text{all } x)$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \ldots + \frac{(-1)^r x^{2r+1}}{(2r+1)!} + \ldots \qquad (\text{all } x)$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \ldots + \frac{(-1)^r x^{2r}}{(2r)!} + \ldots \qquad (\text{all } x)$$

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \ldots + \frac{(-1)^{r+1} x^r}{r} + \ldots \qquad (-1 < x \le 1)$$

Partial fractions decomposition

Non-repeated linear factors:

$$\frac{px+q}{(ax+b)(cx+d)} = \frac{A}{(ax+b)} + \frac{B}{(cx+d)}$$

Repeated linear factors:

$$\frac{px^2+qx+r}{(ax+b)(cx+d)^2} = \frac{A}{(ax+b)} + \frac{B}{(cx+d)} + \frac{C}{(cx+d)^2}$$

Non-repeated quadratic factor:

$$\frac{px^2+qx+r}{(ax+b)(x^2+c^2)} = \frac{A}{(ax+b)} + \frac{Bx+C}{(x^2+c^2)}$$

Trigonometry

$$\sin(A \pm B) \equiv \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) \equiv \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) \equiv \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A \equiv 2 \sin A \cos A$$

$$\cos 2A \equiv \cos^2 A - \sin^2 A \equiv 2\cos^2 A - 1 \equiv 1 - 2\sin^2 A$$

$$\tan 2A \equiv \frac{2\tan A}{1 - \tan^2 A}$$

$$\sin P + \sin Q \equiv 2\sin \tfrac{1}{2}(P+Q)\cos \tfrac{1}{2}(P-Q)$$

$$\sin P - \sin Q \equiv 2\cos \tfrac{1}{2}(P+Q)\sin \tfrac{1}{2}(P-Q)$$

$$\cos P + \cos Q \equiv 2\cos \tfrac{1}{2}(P+Q)\cos \tfrac{1}{2}(P-Q)$$

$$\cos P - \cos Q \equiv -2\sin \tfrac{1}{2}(P+Q)\sin \tfrac{1}{2}(P-Q)$$

Principal values:

$$-\tfrac{1}{2}\pi \ \leq\ \sin^{-1}x \ \leq\ \tfrac{1}{2}\pi \qquad\qquad (|x| \leq 1)$$

$$0 \ \leq\ \cos^{-1}x \ \leq\ \pi \qquad\qquad (|x| \leq 1)$$

$$-\tfrac{1}{2}\pi < \tan^{-1}x < \tfrac{1}{2}\pi$$

Derivatives

f(x)	f′(x)
$\sin^{-1} x$	$\dfrac{1}{\sqrt{1-x^2}}$
$\cos^{-1} x$	$-\dfrac{1}{\sqrt{1-x^2}}$
$\tan^{-1} x$	$\dfrac{1}{1+x^2}$
$\sec x$	$\sec x \tan x$

Integrals

(Arbitrary constants are omitted; a denotes a positive constant.)

$f(x)$	$\int f(x)\,dx$	
$\dfrac{1}{x^2 + a^2}$	$\dfrac{1}{a}\tan^{-1}\left(\dfrac{x}{a}\right)$	
$\dfrac{1}{\sqrt{a^2 - x^2}}$	$\sin^{-1}\left(\dfrac{x}{a}\right)$	$\left(\lvert x\rvert < a\right)$
$\dfrac{1}{x^2 - a^2}$	$\dfrac{1}{2a}\ln\left(\dfrac{x-a}{x+a}\right)$	$(x > a)$
$\dfrac{1}{a^2 - x^2}$	$\dfrac{1}{2a}\ln\left(\dfrac{a+x}{a-x}\right)$	$\left(\lvert x\rvert < a\right)$
$\tan x$	$\ln(\sec x)$	$\left(\lvert x\rvert < \tfrac{1}{2}\pi\right)$
$\cot x$	$\ln(\sin x)$	$(0 < x < \pi)$
$\operatorname{cosec} x$	$-\ln(\operatorname{cosec} x + \cot x)$	$(0 < x < \pi)$
$\sec x$	$\ln(\sec x + \tan x)$	$\left(\lvert x\rvert < \tfrac{1}{2}\pi\right)$

Vectors

The point dividing AB in the ratio $\lambda : \mu$ has position vector $\dfrac{\mu\mathbf{a} + \lambda\mathbf{b}}{\lambda + \mu}$

If A is the point with position vector $\mathbf{a} = a_1\mathbf{i} + a_2\mathbf{j} + a_3\mathbf{k}$ and the direction vector $\mathbf{b}$ is given by $\mathbf{b} = b_1\mathbf{i} + b_2\mathbf{j} + b_3\mathbf{k}$, then the straight line through A with direction vector $\mathbf{b}$ has cartesian equation

$$\frac{x - a_1}{b_1} = \frac{y - a_2}{b_2} = \frac{z - a_3}{b_3} \; (= \lambda)$$

The plane through A with normal vector $\mathbf{n} = n_1\mathbf{i} + n_2\mathbf{j} + n_3\mathbf{k}$ has cartesian equation

$$n_1 x + n_2 y + n_3 z + d = 0 \text{ where } d = -\mathbf{a}.\mathbf{n}$$

Numerical methods

Euler's Method with step size *h*:

$$y_{n+1} = y_n + h\, \text{f}(x_n, y_n)$$

Improved Euler Method with step size *h*:

$$u_{n+1} = y_n + h\, \text{f}(x_n, y_n)$$

$$y_{n+1} = y_n + \frac{h}{2}\left[\text{f}(x_n, y_n) + \text{f}(x_{n+1}, u_{n+1})\right]$$

STATISTICS

Standard discrete distributions

Distribution of X	$\text{P}(X = x)$	Mean	Variance
Binomial $\text{B}(n,p)$	$\binom{n}{x} p^x (1-p)^{n-x}$	np	$np(1-p)$
Poisson $\text{Po}(\lambda)$	$\text{e}^{-\lambda}\dfrac{\lambda^x}{x!}$	λ	λ

Sampling and testing

Unbiased variance estimate from a single sample:

$$s^2 = \frac{1}{n-1}\left(\Sigma x^2 - \frac{(\Sigma x)^2}{n}\right) = \frac{1}{n-1}\Sigma(x - \bar{x})^2$$

Regression and correlation

Estimated product moment correlation coefficient:

$$r = \frac{\Sigma(x-\bar{x})(y-\bar{y})}{\sqrt{\{\Sigma(x-\bar{x})^2\}\{\Sigma(y-\bar{y})^2\}}} = \frac{\Sigma xy - \dfrac{\Sigma x \Sigma y}{n}}{\sqrt{\left(\Sigma x^2 - \dfrac{(\Sigma x)^2}{n}\right)\left(\Sigma y^2 - \dfrac{(\Sigma y)^2}{n}\right)}}$$
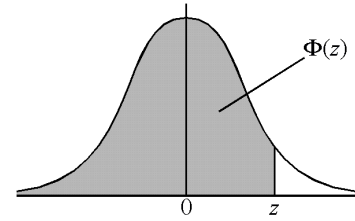
Estimated regression line of *y* on *x*:

$$y - \bar{y} = b(x - \bar{x}), \quad \text{where } b = \frac{\Sigma(x-\bar{x})(y-\bar{y})}{\Sigma(x-\bar{x})^2}$$

THE NORMAL DISTRIBUTION FUNCTION

If Z has a normal distribution with mean 0 and variance 1 then, for each value of z, the table gives the value of $\Phi(z)$, where

$$\Phi(z) = P(Z \le z).$$

For negative values of z use $\Phi(-z) = 1 - \Phi(z)$.

z	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
														ADD					
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359	4	8	12	16	20	24	28	32	36
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753	4	8	12	16	20	24	28	32	36
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141	4	8	12	15	19	23	27	31	35
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517	4	7	11	15	19	22	26	30	34
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879	4	7	11	14	18	22	25	29	32
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224	3	7	10	14	17	20	24	27	31
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549	3	7	10	13	16	19	23	26	29
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852	3	6	9	12	15	18	21	24	27
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133	3	5	8	11	14	16	19	22	25
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389	3	5	8	10	13	15	18	20	23
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621	2	5	7	9	12	14	16	19	21
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830	2	4	6	8	10	12	14	16	18
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015	2	4	6	7	9	11	13	15	17
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177	2	3	5	6	8	10	11	13	14
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319	1	3	4	6	7	8	10	11	13
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441	1	2	4	5	6	7	8	10	11
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545	1	2	3	4	5	6	7	8	9
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633	1	2	3	4	4	5	6	7	8
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706	1	1	2	3	4	4	5	6	6
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767	1	1	2	2	3	4	4	5	5
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817	0	1	1	2	2	3	3	4	4
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857	0	1	1	2	2	2	3	3	4
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890	0	1	1	1	2	2	2	3	3
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916	0	1	1	1	1	2	2	2	2
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936	0	0	1	1	1	1	1	2	2
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952	0	0	0	1	1	1	1	1	1
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964	0	0	0	0	1	1	1	1	1
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974	0	0	0	0	0	1	1	1	1
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981	0	0	0	0	0	0	0	1	1
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986	0	0	0	0	0	0	0	0	0

Critical values for the normal distribution

If Z has a normal distribution with mean 0 and variance 1 then, for each value of p, the table gives the value of z such that
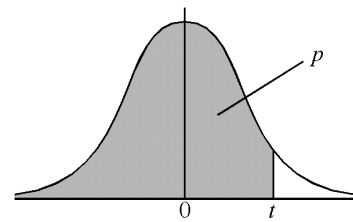
$$P(Z \le z) = p.$$

p	0.75	0.90	0.95	0.975	0.99	0.995	0.9975	0.999	0.9995
z	0.674	1.282	1.645	1.960	2.326	2.576	2.807	3.090	3.291

CRITICAL VALUES FOR THE *t*-DISTRIBUTION

If T has a *t*-distribution with ν degrees of freedom then, for each pair of values of p and ν, the table gives the value of t such that

$$P(T \le t) = p.$$

p	0.75	0.90	0.95	0.975	0.99	0.995	0.9975	0.999	0.9995
$\nu = 1$	1.000	3.078	6.314	12.71	31.82	63.66	127.3	318.3	636.6
2	0.816	1.886	2.920	4.303	6.965	9.925	14.09	22.33	31.60
3	0.765	1.638	2.353	3.182	4.541	5.841	7.453	10.21	12.92
4	0.741	1.533	2.132	2.776	3.747	4.604	5.598	7.173	8.610
5	0.727	1.476	2.015	2.571	3.365	4.032	4.773	5.894	6.869
6	0.718	1.440	1.943	2.447	3.143	3.707	4.317	5.208	5.959
7	0.711	1.415	1.895	2.365	2.998	3.499	4.029	4.785	5.408
8	0.706	1.397	1.860	2.306	2.896	3.355	3.833	4.501	5.041
9	0.703	1.383	1.833	2.262	2.821	3.250	3.690	4.297	4.781
10	0.700	1.372	1.812	2.228	2.764	3.169	3.581	4.144	4.587
11	0.697	1.363	1.796	2.201	2.718	3.106	3.497	4.025	4.437
12	0.695	1.356	1.782	2.179	2.681	3.055	3.428	3.930	4.318
13	0.694	1.350	1.771	2.160	2.650	3.012	3.372	3.852	4.221
14	0.692	1.345	1.761	2.145	2.624	2.977	3.326	3.787	4.140
15	0.691	1.341	1.753	2.131	2.602	2.947	3.286	3.733	4.073
16	0.690	1.337	1.746	2.120	2.583	2.921	3.252	3.686	4.015
17	0.689	1.333	1.740	2.110	2.567	2.898	3.222	3.646	3.965
18	0.688	1.330	1.734	2.101	2.552	2.878	3.197	3.610	3.922
19	0.688	1.328	1.729	2.093	2.539	2.861	3.174	3.579	3.883
20	0.687	1.325	1.725	2.086	2.528	2.845	3.153	3.552	3.850
21	0.686	1.323	1.721	2.080	2.518	2.831	3.135	3.527	3.819
22	0.686	1.321	1.717	2.074	2.508	2.819	3.119	3.505	3.792
23	0.685	1.319	1.714	2.069	2.500	2.807	3.104	3.485	3.768
24	0.685	1.318	1.711	2.064	2.492	2.797	3.091	3.467	3.745
25	0.684	1.316	1.708	2.060	2.485	2.787	3.078	3.450	3.725
26	0.684	1.315	1.706	2.056	2.479	2.779	3.067	3.435	3.707
27	0.684	1.314	1.703	2.052	2.473	2.771	3.057	3.421	3.689
28	0.683	1.313	1.701	2.048	2.467	2.763	3.047	3.408	3.674
29	0.683	1.311	1.699	2.045	2.462	2.756	3.038	3.396	3.660
30	0.683	1.310	1.697	2.042	2.457	2.750	3.030	3.385	3.646
40	0.681	1.303	1.684	2.021	2.423	2.704	2.971	3.307	3.551
60	0.679	1.296	1.671	2.000	2.390	2.660	2.915	3.232	3.460
120	0.677	1.289	1.658	1.980	2.358	2.617	2.860	3.160	3.373
∞	0.674	1.282	1.645	1.960	2.326	2.576	2.807	3.090	3.291

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