# APGP lesson 1

Mr Ian Ang

Math Academy

# Arithmetic Progression

$$1, \quad 6, \quad 11, \quad 16, \quad 21, \quad \ldots$$

$$1, \quad 1+5, \quad 1+(2)5, \quad 1+(3)5, \quad 1+(4)5, \quad \ldots$$

$$a, \quad a+d, \quad a+2d, \quad a+3d, \quad a+4d, \quad \ldots$$

first term : $a$

common difference : $d$

A sequence of the above form is called an **arithmetic progression**, or A.P.

$$T_n = a + (n-1)d$$

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## Example (1)

In an arithmetic progression, the $6^{th}$ term is twice the $3^{rd}$ term and the $10^{th}$ term is 20. Find the common difference in this sequence.

**Solution:**

$T_6 = a + 5d$.

$T_3 = a + 2d$.

Given that $T_6 = 2 \times T_3$,

$$a + 5d = 2(a + 2d)$$
$$a + 5d = 2a + 4d$$
$$d = a \quad \cdots (1)$$

Given that the $10^{th}$ term is 20,

$$a + 9d = 20 \quad \cdots (2)$$

Solving simultaneous equation with (1) and (2), we get $d = 2$.

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## Example (2)

Given that the $4^{th}$ and $8^{th}$ term of an A.P. are 7 and 35 respectively, find the $n^{th}$ term of the sequence. $[a = -14, d = 7; 7n - 21]$

## Example (3)

Find the number of terms in the arithmetic progression $5, 7, 9, \ldots, 97$. $[n = 47]$

$$T_n = a + (n-1)d$$
$$97 = 5 + (n-1)(2)$$
$$97 = 5 + 2n - 2$$
$$94 = 2n$$
$$n = 47$$

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$$S_n = T_1 + T_2 + T_3 + \cdots + T_n.$$

## Sum of A.P.

The sum of the first $n$ terms of an A.P. is given by

$$S_n = \frac{n}{2}(a + \ell).$$

$n$: number of terms
$a$: first term
$\ell$: last term ($n^{th}$ term)

Since $\ell = a + (n-1)d$,

$$S_n = \frac{n}{2}[a + a + (n-1)d]$$

where $d$ is the common difference.

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## Example (4)

An arithmetic progression $G$ has first term $a$ and a non-zero common difference $d$. Given that the $6^{th}$ term is 37 and the sum of the first 10 terms in $G$ is twice the $22^{nd}$ term of the series, find the values of $a$ and $d$. [3]

$$\begin{aligned} 6^{th} \text{ term} &= 37 \\ a + 5d &= 37 \quad \cdots (1) \end{aligned}$$

$$\begin{aligned} \text{Sum of first 10 terms in } G &= 2 \times 22^{nd} \text{ term} \\ \frac{10}{2}[a + (a + 9d)] &= 2(a + 21d) \\ &\vdots \\ a &= -\frac{3}{8}d \quad \cdots (2) \end{aligned}$$

Solving the simultaneous equation using (1) and (2), we get:

$$d = 8, \quad a = -3.$$

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A new series $H$ is formed by selecting every third term of $G$. Find the sum of the first 50 terms of $H$.

Sum of first 50 terms in $H$

$= (a + 2d) + (a + 5d) + (a + 8d) + \cdots + 50^{th}$ term

$=$ Sum of A.P. with first term: $a + 2d = 13$ and common difference:$3d = 24$

$= \frac{50}{2}[13 \quad + \quad 13 + (50 - 1)(24)]$

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Work on the next 2 examples and take a short break.

## Example (5)

Find the sum of all the odd numbers from 3 to 99 inclusive which are not multiples of 5.
(Hint: Find separately, the sum of the odd numbers AND those that are multiples of 5.)

$3 + 5 + \ldots + 99$
Number of terms:

$$3 + (n - 1)(2) = 99$$
$$n = 49$$

Total sum: $\frac{49}{2}[3 + 99] = 2499$
$5 + 15 + \ldots + 95$
Number of terms:

$$5 + (n - 1)(10) = 95$$
$$n = 10$$

Total sum: $\frac{10}{2}[5 + 95] = 500$
Required sum $=$ 2499 - 500 $= 1999$

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(Hint: Find separately, the sum of the odd numbers AND those that are multiples of 5.)

$3 + 5 + \ldots + 99$
Number of terms:

$$\begin{aligned} 3 + (n-1)(2) &= 99 \\ n &= 49 \end{aligned}$$

Total sum: $\frac{49}{2}[3 + 99] = 2499$
$5 + 15 + \ldots + 95$
Number of terms:

$$\begin{aligned} 5 + (n-1)(10) &= 95 \\ n &= 10 \end{aligned}$$

Total sum: $\frac{10}{2}[5 + 95] = 500$
Required sum $=$ 2499 - 500 $=$ 1999

## Example (6)

The $9^{th}$ term of an arithmetic progression is 50 and the sum of the first 15 terms is 570. It is given that the sum of the first $n$ terms is greater than 500. Find the least possible value of $n$.

$$T_9 : \quad a + 8d = 50 \ldots (1)$$

$$\begin{aligned} S_{15} : \quad \frac{15}{2}\left[a + a + 14d\right] &= 570 \\ \frac{15}{2}\left[2a + 14d\right] &= 570 \\ 15a + 105d &= 570 \ldots (2) \end{aligned}$$

Solving for simultaneous equations in (1) and (2), we obtain $a = -46$ and $d = 12$.

$$\begin{aligned} S_n &> 500 \\ \frac{n}{2}\left[a + a + (n-1)d\right] &> 500 \\ \frac{n}{2}\left[-46 - 46 + (n-1)(12)\right] &> 500 \\ \frac{n}{2}\left[-92 + 12n - 12\right] &> 500 \\ \frac{n}{2}\left[-104 + 12n\right] &> 500 \\ -104n + 12n^2 &> 1000 \\ 12n^2 - 104n - 1000 &> 0 \\ n < -5.77(\text{rej}) \text{ or } n &> 14.44 \end{aligned}$$

Therefore, least $n$ is 15.

$$S_n = T_1 + T_2 + \cdots + T_{n-1} + T_n$$
$$S_{n-1} = T_1 + T_2 + \cdots + T_{n-1}$$

Finding $T_n$ fron $S_n$

$$T_n = S_n - S_{n-1}$$

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Finding $T_n$ fron $S_n$

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## Example (7)

The sum of the first $n$ terms of a series is given by $S_n = n(4+n)$. Find the general formula for the $n^{th}$ term.

$$\begin{aligned} T_n &= S_n - S_{n-1} \\ &= n(4+n) - (n-1)(4+n-1) \\ &= 4n + n^2 - (n-1)(n+3) \\ &= 4n + n^2 - (n^2 + 2n - 3) \\ &= 4n + n^2 - n^2 - 2n + 3 \\ &= 2n + 3 \end{aligned}$$

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## Example (7)

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## Prove that a sequence is an A.P.

We show that the common difference is a constant, i.e,

$$T_n - T_{n-1} = d$$

### Example (8)

The sum to $n$ terms of a series is $S_n = 8n^2 - 4n$. Calculate the $n^{th}$ term. Hence prove that the sequence is an arithmetic progression.

$$\begin{aligned} T_n &= S_n - S_{n-1} \\ &= 8n^2 - 4n - [8(n-1)^2 - 4(n-1)] \\ &\vdots \\ &= 16n - 12 \end{aligned}$$

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