APGP Tutorial 1

1. [2011/TJC/I/9a]

   Find the sum of all the integers between 1102 and 2011 (both inclusive) which are not divisible by 3. [5]

   [944644]

2. [2010/ACJC/I/10b]

   The terms $u_1, u_2, u_3, \ldots$ form an arithmetic sequence with first term a and having non-zero common difference d.
   Given that the sum of the first 10 terms of the sequence is 105 more than $10u_5$, find the common difference.
   If u_{26} is the first term in the sequence which is greater than 542, find the range of values of a. [6]

   [$d = 21$; $17 < a \leq 38$]

3. [2012/TJC/Prelim/I/7a]

   A finite arithmetic progression has n terms and common difference d. The first term is 1 and the sum of the last 5 terms exceeds the sum of the first 4 terms by 193.

   (a) Show that $5nd - 21d - 192 = 0$. [3]

   (b) Given also that the 6^{th} term of the progression is 16, find n. [2]

   [(b) 17]

4. [2011/MI/I/13a]

   The sum of the first n terms, S_n of a sequence is given by

   $$S_n = 2n^2 + kn - 3,$$

   where k is a constant.
   By finding the n^{th} term of the sequence, prove that the sequence is an arithmetic progression.
   [$T_n = 4n + k - 2$; $T_n - T_{n-1} = 4$] [5]

5. [2009/DHS/Y6CT/8a]

   The sum, S_n, of the first n terms of a sequence is given by $S_n = 3n^2$, $n \geq 1$.

   (a) Show that this sequence is an arithmetic progression. [2]

   (b) Find the least value of n for which S_n exceeds 244. [2]

   [(b) 10]

6. [2015/TJC/Prelim/II/2a]

   Given that the sequence $5, 11, 17, \ldots, x$ is arithmetic, solve the equation

   $$5 + 11 + 17 + \cdots + x = 2760.$$

   [4]

   [$x = 179$]

7. [2010/YJC/I/8b]

   Adam has 6643 marbles that he wants to put in boxes.

   If he puts 13 marbles in the first box and for each subsequent box, he puts 13 marbles more than what he puts in the previous box, how many boxes will he need and what is the number of marbles in the last box? [4]

   [195marbles, 32boxes]

8. [2010/RI/I/14b]

An arithmetic progression is grouped into sets of numbers as follows:

$$\{2\}, \{6, 10\}, \{14, 18, 22\}, \{26, 30, 34, 38\}, \ldots$$

where the number of terms is 1 for the first set, 2 for the second set, 3 for the third set, 4 for the fourth set and so forth.

(a) Find the total number of terms in the first n sets.

(b) Show that the first term in the n^{th} set is $2n^2 - 2n + 2$.

(c) Find, in terms of n, a simplified expression for the last term in the n^{th} set.

(d) Hence, find the sum of all the terms in the n^{th} set.

[(a) $\frac{n}{2}(1 + n)$ (c)$2n^2 + 2n - 2$, (d)$2n^3$]

