APGP Tutorial 1: Solutions

1. Strategy: We will find the sum of all terms from 1102 to 2011, subtract by sum all the terms that are divisible by 3.

   From 1102 to 2011,

   <div align="center">

   First term that is divisible by 3:   1104

   Last term that is divisible by 3:   2010

   </div>

   Note that the terms
   $$1104, 1107, 1110, \ldots, 2007, 2010$$
   forms an AP with 1st term 1104, common difference 3. First, find the number of terms that are divisible by 3.

   $$a + (n-1)d = 2010$$
   $$1103 + (n-1)(3) = 2010$$
   $$n = 303$$

   Sum of terms divisible by 3: $\frac{303}{2}(1104 + 2010) = 471771$

   Number of terms from 1102 to 2011: $2011 - 1102 + 1 = 910$

   Sum of all terms: $\frac{910}{2}(1102 + 2011) = 1416415$

   Sum required: $1416415 - 471771 = 944644$

2.

   $$S_{10} = 105 + 10u_5$$
   $$\frac{10}{2}(a + a + 9d) = 105 + 10(a + 4d)$$
   $$5(2a + 9d) = 105 + 10a + 40d$$
   $$10a + 45d = 105 + 10a + 40d$$
   $$5d = 105$$
   $$d = 21$$

   $$
   \begin{aligned}
   u_{26} &> 542 & u_{25} &\leq 542 \\
   a + 25d &> 542 & a + 24d &\leq 542 \\
   a + 25(21) &> 542 & a + 24(21) &\leq 542 \\
   a &> 17 & a &\leq 38
   \end{aligned}
   $$

   $\therefore 17 < a \leq 38$.

3.

Sum of last 5 terms $= u_{n-4} + u_{n-3} + u_{n-2} + u_{n-1} + u_n$

$$= \frac{5}{2}(u_{n-4} + u_n)$$

$$= \frac{5}{2}[1 + (n-5)d + 1 + (n-1)d]$$

$$= \frac{5}{2}[2 + 2nd - 6d]$$

$$= 5 + 5nd - 15d$$

Sum of first 4 terms $= \frac{4}{2}(1 + 1 + 3d)$

$$= 2(2 + 3d)$$

$$= 4 + 6d$$

Sum of last 5 terms $=$ Sum of first 4 terms $+ 193$

$$5 + 5nd - 15d = 4 + 6d + 193$$

$$5nd - 21d - 192 = 0 \text{ (shown)}$$

$$6^{th} \text{ term} = 16$$

$$1 + 5d = 16$$

$$5d = 15$$

$$d = 3$$

$$\therefore 5n(3) - 21(3) - 192 = 0$$

$$n = 17$$

4.

$$T_n = S_n - S_{n-1}$$

$$= 2n^2 + kn - 3 - [2(n-1)^2 + k(n-1) - 3]$$

$$= 2n^2 + kn - 3 - [2n^2 - 4n + 2 + kn - k - 3]$$

$$= 2n^2 + kn - 3 - [2n^2 + nk - 4n - k - 1]$$

$$= 4n + k - 2$$

$$T_n - T_{n-1} = 4n + k - 2 - [4(n-1) + k - 2]$$

$$= 4n + k - 2 - 4n + 4 - k + 2$$

$$= 4$$

Since $T_n - T_{n-1}$ is a constant, the sequence is an A.P.

5. (a)

$$T_n = S_n - S_{n-1}$$

$$= 3n^2 - 3(n-1)^2$$

$$= 3n^2 - 3(n^2 - 2n + 1)$$

$$= 3n^2 - 3n^2 + 6n - 3$$

$$= 6n - 3$$

$$T_n - T_{n-1} = 6n - 3 - [6(n-1) - 3]$$

$$= 6n - 3 - (6n - 9)$$

$$= -3 + 9$$

$$= 6$$

Since $T_n - T_{n-1}$ is a constant, the sequence is an A.P.

(b)

$$3n^2 > 244$$
$$n^2 > 81.333$$
$$(n - 9.018)(n + 9.018) > 0$$
$$n > 9.018 \text{ or } n < -9.018(\text{rej})$$

$$\therefore \text{ least } n = 10.$$

6.

$$\frac{n}{2}(5 + 5 + (n-1)(6)) = 2760$$
$$3n^2 + 2n - 2760 = 0$$
$$n = 30 \text{ or } n = \frac{-92}{3}(\text{rej as } n > 0)$$

$$\therefore x = T_{30} = 5 + 29(6) = 179$$

7. Let n be the number of boxes that Adam requires.

$$\text{Total number of all marbles in } n \text{ boxes} = 6643$$
$$\frac{n}{2}[13 + 13 + (n-1)(13)] = 6643$$
$$\frac{n}{2}[13n + 13] = 6643$$
$$13n^2 + 13n - 13286 = 0$$
$$n = 31.473 \text{ or } -32.473$$

Hence, he will required 32 boxes.

We now count how many marbes there are in the first 31 boxes.

$$\frac{31}{2}(13 + 13 + 30(13)) = 6448$$

Therefore, number of marbles in the last box is 6643 - 6448 = 195.

8. (a) 1st set: 1 term
   2nd set: 2 terms
   3rd set: 3 terms
   $\vdots$
   nth set: n terms

   Total number of terms $= 1 + 2 + 3 + \cdots + n = \frac{n}{2}(1 + n)$

(b)

Set	Number of Terms so far	First term of the Set	Last term of the set
1	1	T_1	T_1
2	1+2 = 3	T_2	T_3
3	1+2 +3 = 6	T_4	T_6
4	1+2 +3 +4 =10	T_7	T_{10}
5	1+2 +3 +4 +5=15	T_{11}	T_{15}
$n-1$			
n			

Using the results from (a), total number of terms in the first $(n-1)$ sets $= \frac{n-1}{2}(n)$. Therefore, the term corresponding to the first term in the nth set is the $\left(\frac{n-1}{2}(n)+1\right)^{th}$ term.

$$T_{\frac{n-1}{2}(n)+1} = 2 + \left[\left(\frac{n-1}{2}(n)+1\right)-1\right](4) \qquad \text{(Using AP formula } T_n = a + (n-1)d)$$
$$= 2n^2 - 2n + 2$$

(c) Total number of terms in the first n sets $= \frac{n}{2}(1+n)$. Therefore, the term corresponding to the last term in the nth set is the $\left(\frac{n}{2}(1+n)\right)^{th}$ term.

$$T_{\frac{n}{2}(1+n)} = 2 + \left[\frac{n}{2}(1+n)-1\right](4) \qquad \text{(Using AP formula } T_n = a + (n-1)d)$$
$$= 2n^2 + 2n - 2$$

(d) First term in n^{th} set: $2n^2 - 2n + 2$
Last term in n^{th} set: $2n^2 + 2n - 2$

$$\text{Sum of all terms in } n^{th} \text{ set} = \frac{n}{2}[(2n^2 - 2n + 2) + 2n^2 + 2n - 2] = 2n^3$$