Differential Equation Tutorial 1

1. [2010/IJC/Prelim/I/6]

   (a) Show, by means of substitution $w = x^2y$, that the differential equation

   $$x\frac{dy}{dx} + 2y + 3xy = 0$$

   can be reduced to the form

   $$\frac{dw}{dx} = -3w$$

   [2]

   (b) Hence find y in terms of x, given that $y = -\frac{1}{2}$ when $x = 2$. [4]

   [(b) $y = -\frac{2}{x^2}e^{6-3x}$]

2. [2010/NJC/Prelim/I/6]

   (a) By using the substitution $y = vx$, find the general solution of the differential equation

   $$x\frac{dy}{dx} = 3x + y - 2$$

   [4]

   (b) State the equation of the locus where the stationary points of the solution curves lie. [1]

   (c) Sketch, on a single diagram, the graph of the locus found in part(b) and two members of the family of solution curves where the arbitrary constant in the solution is non-zero. [3]

   [$y = 3x\ln|x| + 2 + Cx$ (a) $y = -3x + 2$]

3. [2010/JJC/Prelim/II/2]

   (a) A particular solution of a differential equation is given by $(x+y)^2 = 2xy - \frac{2}{3}y^3$. Show that

   $$(y^2 + y)\frac{dy}{dx} = -x$$

   [2]

   (b) A second, related, family or curves is given by the differential equation

   $$x\frac{dy}{dx} = y^2 + y$$

   By means of the substitution $y = ux$, show that the general solution for y, in terms of x, is

   $$y = \frac{-x}{x+c},$$

   where c is an arbitrary constant. [3]

   (c) Sketch, on a single diagram, three distinct members of the second family of solution curves, stating clearly the coordinates of the points where the curves cross the axes and the equations of any asymptotes. [5]

4. [2010/CJC/Prelim/II/4a,b]

   (a) Verify that $y = x$ is a particular solution of the differential equation

   $$\frac{dy}{dx} = \frac{x^2 + y^2}{2xy},\ x, y \neq 0$$

   [2]

(b) Show that the substitution $y = ux$ reduces the differential equation

$$\frac{dy}{dx} = \frac{x^2 + y^2}{2xy}$$

to the differential equation

$$x\frac{du}{dx} = \frac{1 - u^2}{2u}.$$

Hence find the general solution of the differential equation

$$\frac{dy}{dx} = \frac{x^2 + y^2}{2xy}.$$

[3]

(c) i. Due to a rapid disease outbreak, the population of fish in a river, x (in thousands), is believed to obey the differential equation

$$\frac{d^2x}{dt^2} = 4ae^{-2t}$$

where t is the time in days, and $a > 0$ is a constant. Given that the entire population of fish is wiped out by the disease eventually, show that the general solution of the differential equation is $x = ae^{-2t}$. [3]

ii. Explain the meaning of a, in the context of the question. Sketch the family of solution curves of the differential equation for $a = 1$ and 2. [2]

[b $y^2 = x^2 - Ax$ (c)(ii) a represents the initial population of the fish (in thousands)]

5. [2017/AJC/I/2]

Show that the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} + \frac{3xy}{1 - 3x^2} - x + 1 = 0$$

may be reduced by means of the substitution $y = u\sqrt{1 - 3x^2}$ to

$$\frac{\mathrm{d}u}{\mathrm{d}x} = \frac{x - 1}{\sqrt{1 - 3x^2}}$$

Hence find the general solution for y in terms of x. [5]

[$y = -\frac{1}{3}(1 - 3x^2) - \frac{\sqrt{1-3x^2}}{\sqrt{3}}\sin^{-1}(\sqrt{3}x) + C\sqrt{1 - 3x^2}$]

6. [2017/PJC/II/2]

By differentiating $\cos\frac{\mathrm{d}y}{\mathrm{d}x}$ with respect to x, solve the differential equation

$$\cos x\frac{\mathrm{d}^2y}{\mathrm{d}x^2} - \sin x\frac{\mathrm{d}y}{\mathrm{d}x} = \sec^2 x + \cos 2x,$$

giving y in terms of x. [6]

[$y = \sec x - \cos x + C\ln|\sec x + \tan x| + D$]

