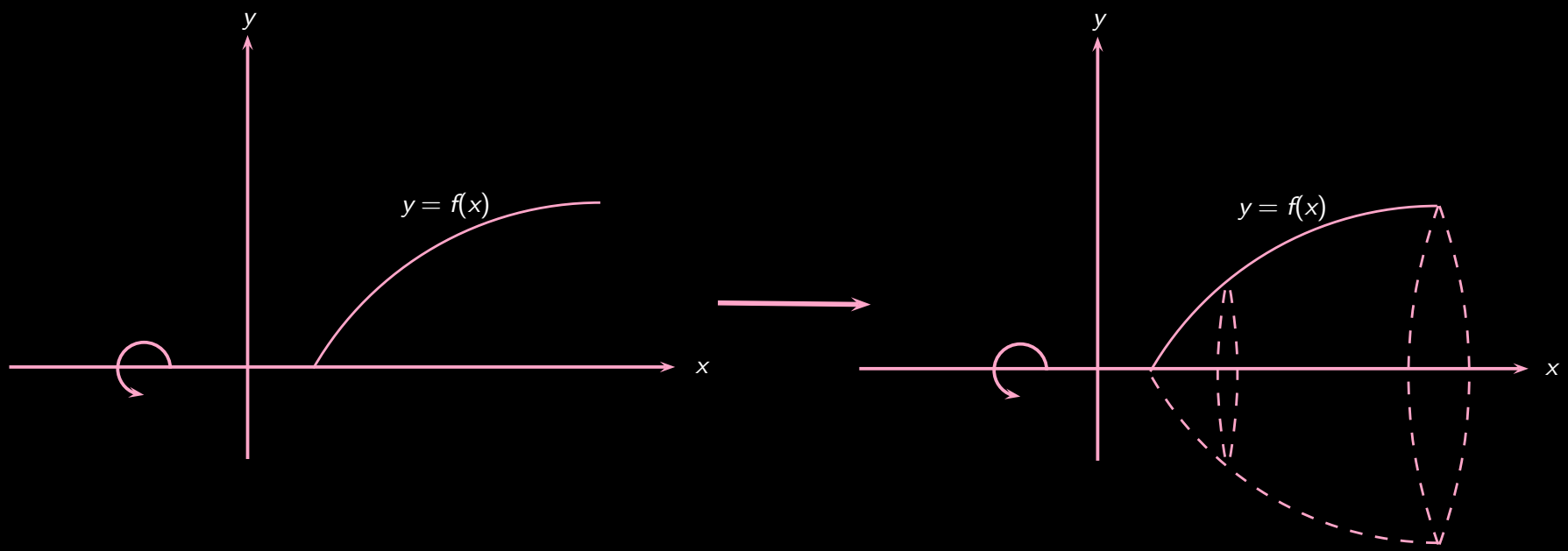
Integration Application: Lesson 2

Mr Ian Ang

Math Academy

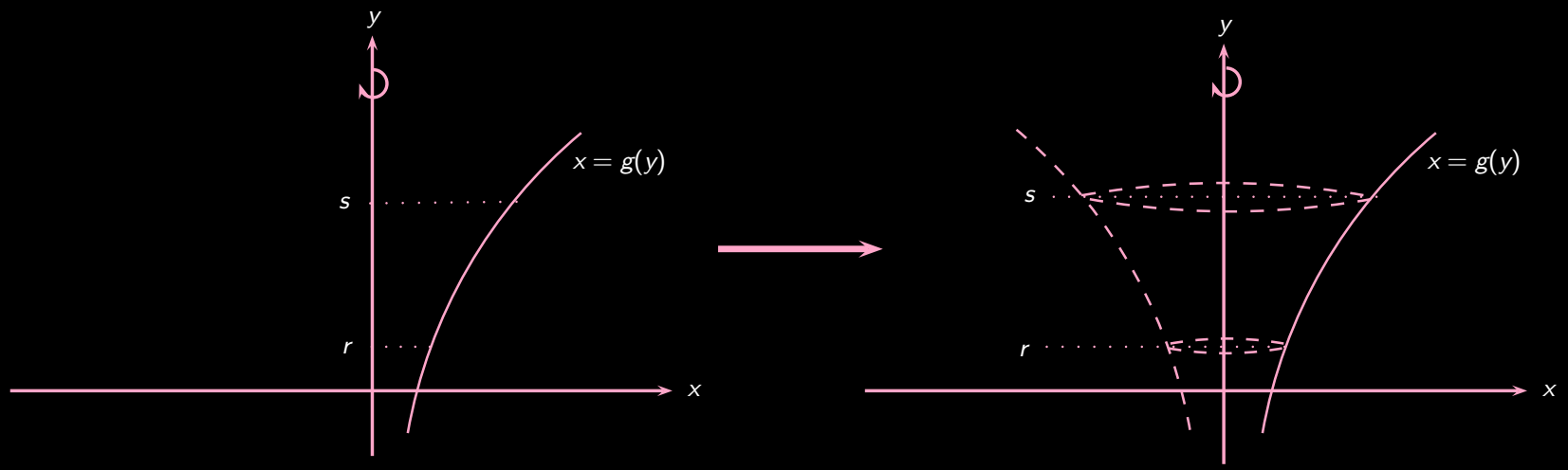
Volume of Revolution



Rotation about x-axis

Volume of revolution of $y = f(x)$ rotated through 360° about the x-axis, from $x = a$ to $x = b$.

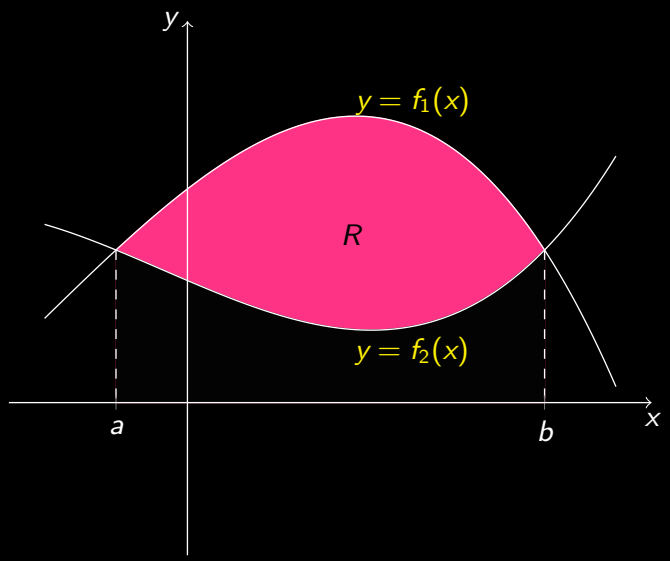
$$V = \pi \int_a^b y^2 \, dx$$

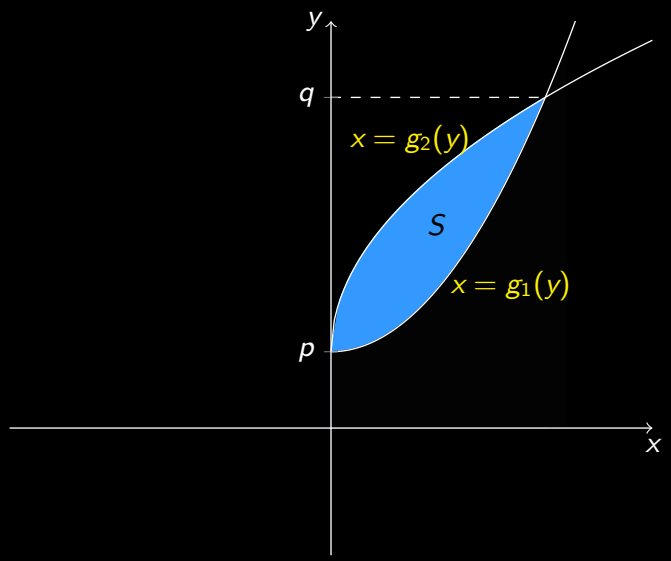


Rotation about y-axis

Volume of revolution of $x = g(y)$ rotated through 360° about the y-axis, from $y = r$ to $y = s$.

$$V = \pi \int_r^s x^2 \, dy$$





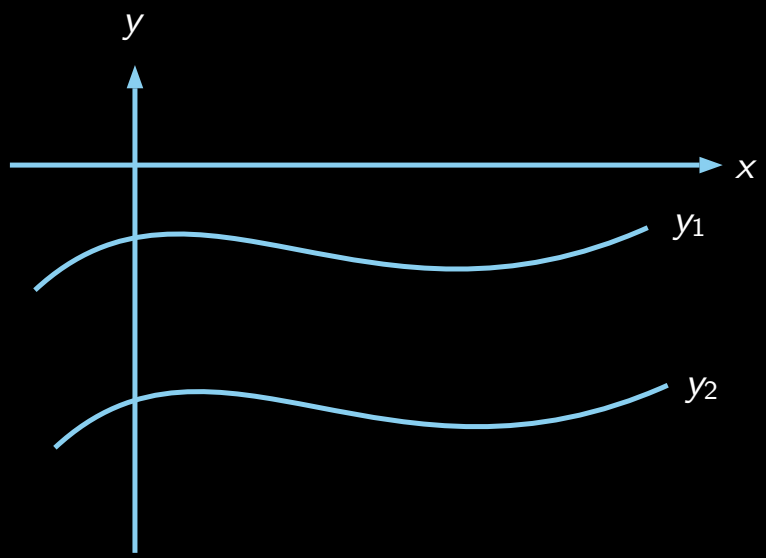
Rotation of Region Bounded by 2 Curves

$$V_R = \pi \int_a^b [f_1(x)]^2 - [f_2(x)]^2 \, dx$$

$$V_S = \pi \int_p^q [g_1(y)]^2 - [g_2(y)]^2 \, dy$$

Always use ‘outer curve’ - ‘inner curve’.

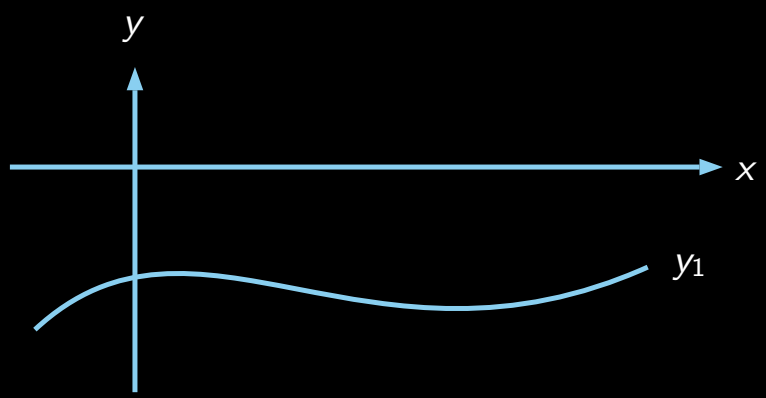
What is the difference between finding area and volume of revolution in the following scenarios?



Area between y_1 and y_2 is given by:

Volume of revolution generated by the region between y_1 and y_2, when rotated 360° about the x-axis is given by:

What is the difference between finding area and volume of revolution in the following scenarios?



Area between y_1 and the x-axis is given by:

Volume of revolution generated by the region between y_1 and x-axis, when rotated 360° about the x-axis is given by:

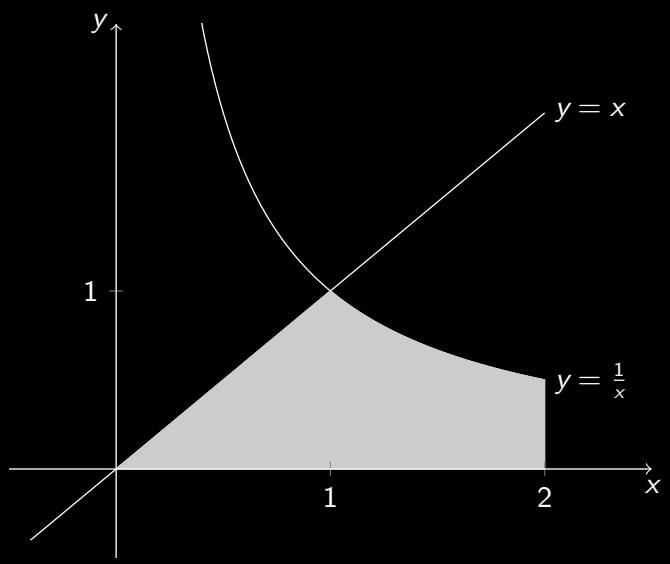
Example (7)

For each of the diagrams below, find the volume of the solid generated by rotating the shaded region through 360°, about the

(i) x-axis,

(ii) y-axis,

(a)



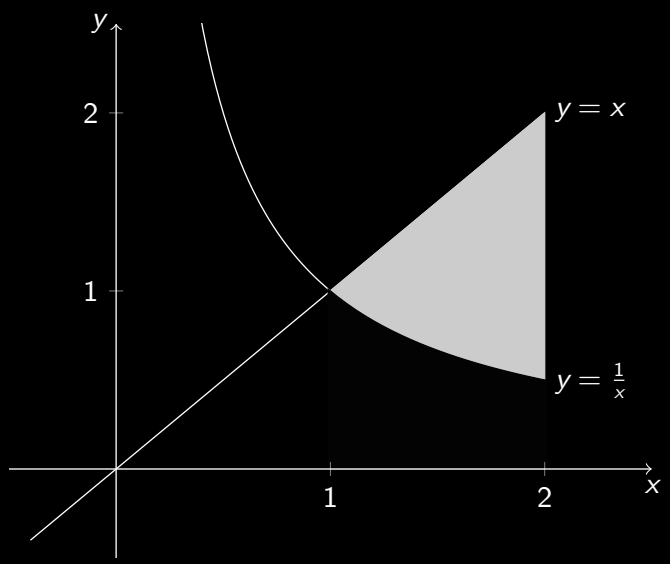
Example (7)

For each of the diagrams below, find the volume of the solid generated by rotating the shaded region through 360°, about the

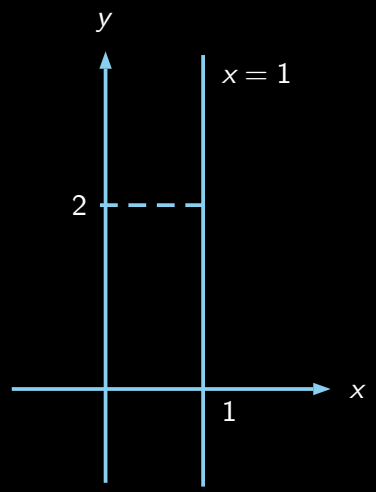
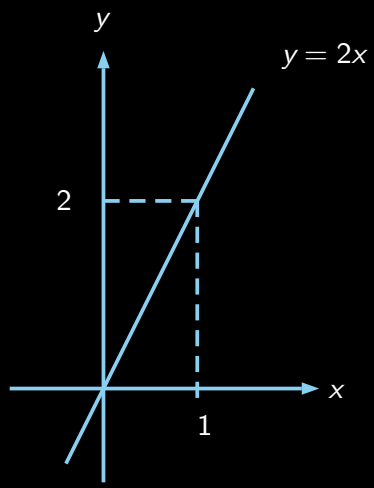
(i) x-axis,

(ii) y-axis,

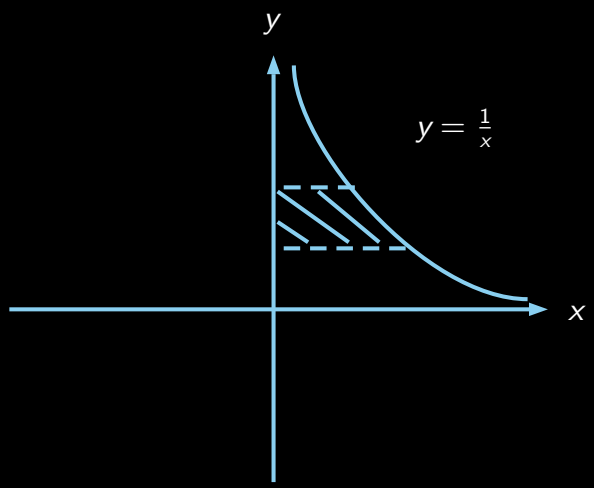
(b)



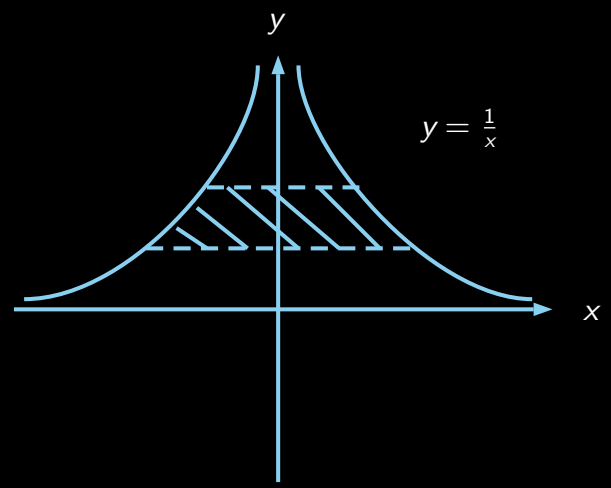
Cones and Cylinders

	Using Integration	Using Formula
		
		

Rotating 180° vs Rotating 360°



Find the volume of the solid formed when the shaded region is rotated through 360° about the y-axis.

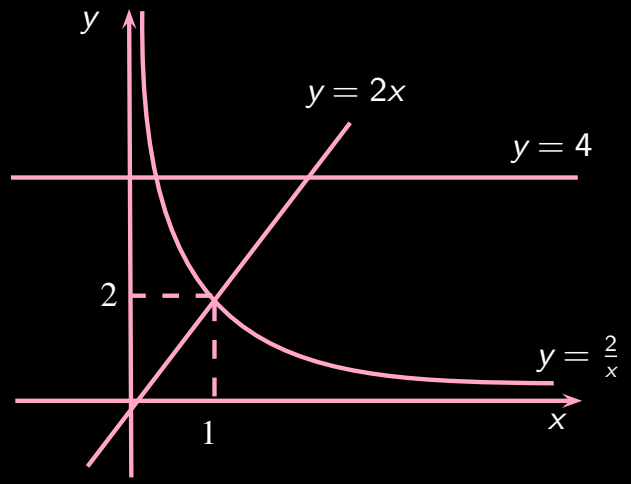


Find the volume of the solid formed when the shaded region is rotated through 180° about the y-axis.

Example (8)

Sketch the curve $y = \frac{2}{x}$ for $x > 0$.
The area enclosed by the curve $y = \frac{2}{x}$, the lines $y = 2x$, $y = 4$ and the y-axis is rotated through 360° about the y-axis. Find the volume generated, giving your answer in terms of π.

$[\frac{5}{3}\pi]$



Required integral $= \pi \int_0^2 \left(\frac{y}{2}\right)^2 \, dy + \pi \int_2^4 \left(\frac{2}{y}\right)^2 \, dy$

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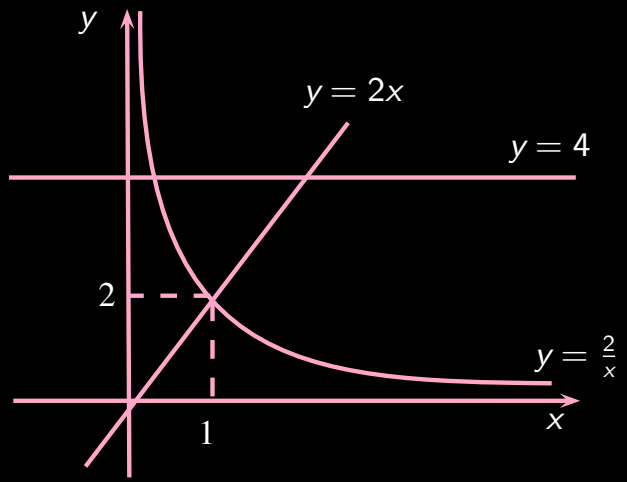

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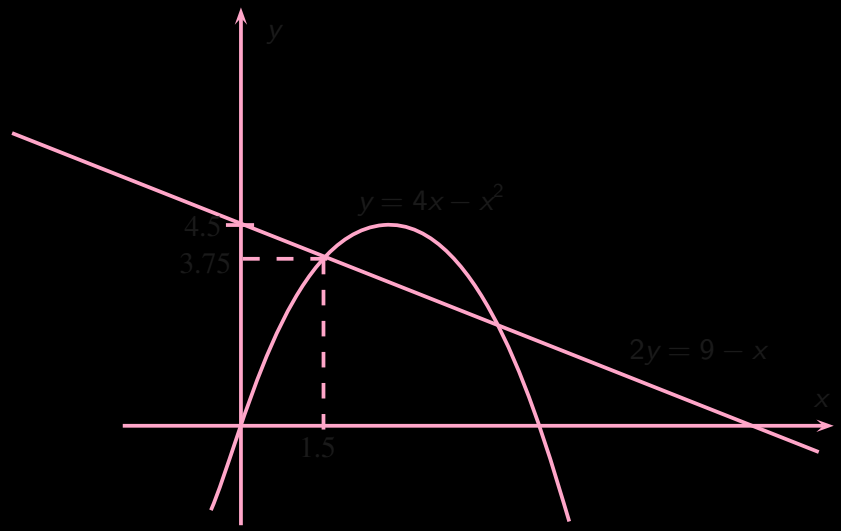
Example (2011/PJC/Prelim/I/1b modified)

The region R is bounded by the curve $y = 4x - x^2$, the line $2y = 9 - x$, and the y-axis. Find the volume of the solid when

(a) R is rotated completely about the x-axis, [3]

(b) R is rotated completely about the y-axis, [3]

Try this!



Using GC, the point of intersection between the 2 graphs is (1.5, 3.75).

(a)

$$\text{Required integral} = \pi \int_0^{1.5} \left[\left(\frac{9-x}{2} \right)^2 - (4x - x^2)^2 \right] dx$$

$$= 50.89 \quad \text{[Using GC]}$$

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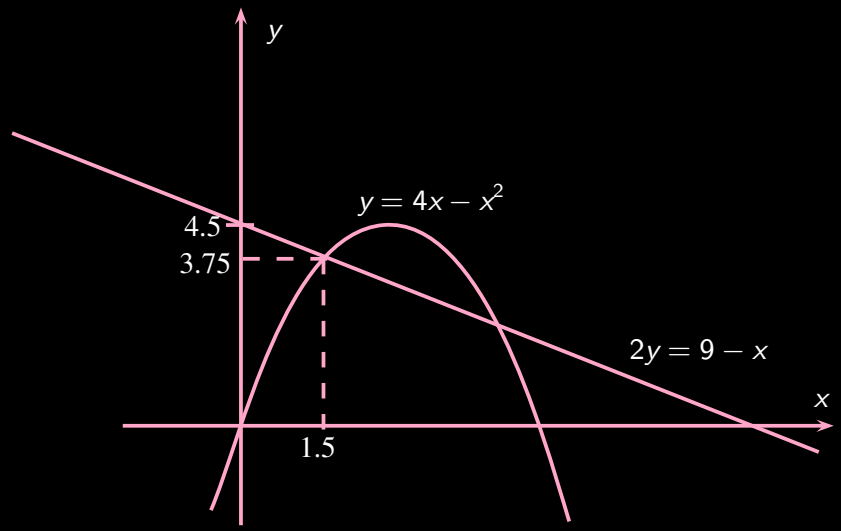

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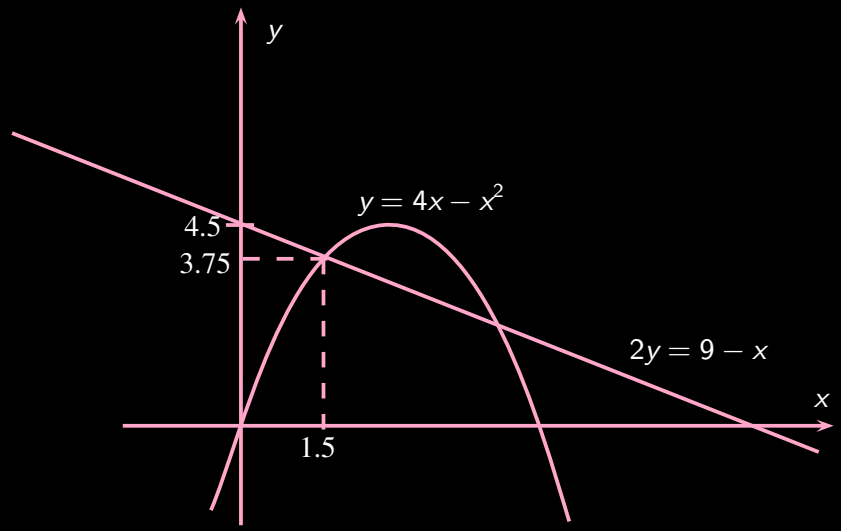

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(b) From $y = 4x - x^2$, we make x the subject.

$$\begin{aligned} y &= 4x - x^2 \\ &= -(x^2 - 4x) \\ &= -[(x-2)^2 - 2^2] \\ &= 4 - (x-2)^2 \\ (x-2)^2 &= 4 - y \\ x &= 2 \pm \sqrt{4-y} \end{aligned}$$

$$\text{Required integral} = \pi \int_0^{3.75} (2 - \sqrt{4-y})^2 \, dy + \pi \int_{3.75}^{4.5} (9 - 2y)^2 \, dy$$

$$= 7.95 \quad \text{[Using GC]}$$

September Revision Classes - Next 5 weeks

- Weekly tests - To confirm your understanding and presentation
- Revision worksheet - Each chapter consists of questions of different types, for your exposure to exam-styled questions
- I also encourage you to ask questions from your school, if you have any

Week / School	PJC
1	Vectors
2	Graphing + Inequalities
3	Differentiation
4	Integration
5	Functions

- Classes will continue as usual in October and November, where we will be starting on new chapters.
- No lessons for the whole of December. We resume in January 2019.

Aug / September Revision Classes [Monday] - Next 6 weeks

- Weekly tests - To confirm your understanding and presentation
- Revision worksheet - Each chapter consists of questions of different types, for your exposure to exam-styled questions
- I also encourage you to ask questions from your school, if you have any

Week / School	PJC	SAJC
1	Vectors	Vectors
2	Integration	Graphing
3	Differentiation	Differentiation
4	Graphing + Inequalities	Sequences and Series
5	Functions	Functions + Inequalities
6	Vectors	Vectors

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