

APGP lesson 1

Mr Ian Ang

Math Academy

Arithmetic Progression

1, 6, 11, 16, 21, ...

1, $1 + 5$, $1 + (2)5$, $1 + (3)5$, $1 + (4)5$, ...

a , $a + d$, $a + 2d$, $a + 3d$, $a + 4d$, ...

first term : a

common difference : d

A sequence of the above form is called an **arithmetic progression**, or A.P.

$$T_n = a + (n - 1)d$$

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Example (1)

In an arithmetic progression, the 6th term is twice the 3rd term and the 10th term is 20. Find the common difference in this sequence.

Solution:

$$T_6 = a + 5d.$$

$$T_3 = a + 2d.$$

Given that $T_6 = 2 \times T_3$,

$$a + 5d = 2(a + 2d)$$

$$a + 5d = 2a + 4d$$

$$d = a \quad \dots (1)$$

Given that the 10th term is 20,

$$a + 9d = 20 \quad \dots (2)$$

Solving simultaneous equation with (1) and (2), we get $d = 2$.

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Example (2)

Given that the 4th and 8th term of an A.P. are 7 and 35 respectively, find the n^{th} term of the sequence. [$a = -14, d = 7; 7n - 21$]

Example (3)

Find the number of terms in the arithmetic progression 5, 7, 9, ..., 97. [$n = 47$]

$$T_n = a + (n - 1)d$$

$$97 = 5 + (n - 1)(2)$$

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$$94 = 2n$$

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Sum of Arithmetic Progression

$$S_n = T_1 + T_2 + T_3 + \cdots + T_n.$$

Sum of A.P.

The sum of the first n terms of an A.P. is given by

$$S_n = \frac{n}{2}(a + \ell).$$

n : number of terms

a : first term

ℓ : last term (n^{th} term)

Since $\ell = a + (n - 1)d$,

$$S_n = \frac{n}{2} [a + a + (n - 1)d]$$

where d is the common difference.

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Example (4)

An arithmetic progression G has first term a and a non-zero common difference d . Given that the 6th term is 37 and the sum of the first 10 terms in G is twice the 22nd term of the series, find the values of a and d . [3]

$$6^{\text{th}} \text{ term} = 37$$

$$a + 5d = 37 \quad \dots (1)$$

Sum of first 10 terms in $G = 2 \times 22^{\text{nd}}$ term

$$\frac{10}{2} [a + (a + 9d)] = 2(a + 21d)$$

$$\vdots$$

$$a = -\frac{3}{8}d \quad \dots (2)$$

Solving the simultaneous equation using (1) and (2), we get:

$$d = 8, \quad a = -3.$$

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A new series H is formed by selecting every third term of G . Find the sum of the first 50 terms of H .

Sum of first 50 terms in H

$$\begin{aligned} &= (a + 2d) + (a + 5d) + (a + 8d) + \cdots + 50^{\text{th}} \text{ term} \\ &= \text{Sum of A.P. with first term: } a + 2d = 13 \text{ and common difference: } 3d = 24 \\ &= \frac{50}{2} [13 + 13 + (50 - 1)(24)] \\ &= 30050 \end{aligned}$$

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Work on the next 2 examples and take a short break.

Example (5)

Find the sum of all the odd numbers from 3 to 99 inclusive which are not multiples of 5.

(Hint: Find separately, the sum of the odd numbers AND those that are multiples of 5.)

$$3 + 7 + 11 + \dots + 99$$

Number of terms:

$$3 + (n - 1)(2) = 99$$

$$n = 49$$

$$\text{Total sum: } \frac{49}{2} [3 + 99] = 2499$$

$$5 + 15 + 25 + \dots + 95$$

Number of terms:

$$5 + (n - 1)(10) = 95$$

$$n = 10$$

$$\text{Total sum: } \frac{10}{2} [5 + 95] = 500$$

$$\text{Required sum} = 2499 - 500 = 1999$$

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Example (6)

The 9th term of an arithmetic progression is 50 and the sum of the first 15 terms is 570. It is given that the sum of the first n terms is greater than 500. Find the least possible value of n .

$$T_9 : a + 8d = 50 \dots (1)$$

$$S_{15} : \frac{15}{2} [a + a + 14d] = 570$$

$$\frac{15}{2} [2a + 14d] = 570$$

$$15a + 105d = 570 \dots (2)$$

Solving for simultaneous equations in (1) and (2), we obtain $a = -46$ and $d = 12$.

$$S_n > 500$$

$$\frac{n}{2} [a + a + (n-1)d] > 500$$

$$\frac{n}{2} [-46 - 46 + (n-1)(12)] > 500$$

$$\frac{n}{2} [-92 + 12n - 12] > 500$$

$$\frac{n}{2} [-104 + 12n] > 500$$

$$-104n + 12n^2 > 1000$$

$$12n^2 - 104n - 1000 > 0$$

$$n < -5.77(\text{rej}) \text{ or } n > 14.44$$

Therefore, least n is 15.

$$S_n = T_1 + T_2 + \cdots + T_{n-1} + T_n$$
$$S_{n-1} = T_1 + T_2 + \cdots + T_{n-1}$$

Finding T_n from S_n

$$T_n = S_n - S_{n-1}$$

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Example (7)

The sum of the first n terms of a series is given by $S_n = n(4 + n)$. Find the general formula for the n^{th} term.

$$\begin{aligned}T_n &= S_n - S_{n-1} \\&= n(4 + n) - (n - 1)(4 + n - 1) \\&= 4n + n^2 - (n - 1)(n + 3) \\&= 4n + n^2 - (n^2 + 2n - 3) \\&= 4n + n^2 - n^2 - 2n + 3 \\&= 2n + 3\end{aligned}$$

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Example (7)

The sum of the first n terms of a series is given by $S_n = n(4 + n)$. Find the general formula for the n^{th} term.

$$\begin{aligned}T_n &= S_n - S_{n-1} \\&= n(4 + n) - (n - 1)(4 + n - 1) \\&= 4n + n^2 - (n - 1)(n + 3) \\&= 4n + n^2 - (n^2 + 2n - 3) \\&= 4n + n^2 - n^2 - 2n + 3 \\&= 2n + 3\end{aligned}$$

Prove that a sequence is an A.P.

We show that the common difference is a constant, i.e.,

$$T_n - T_{n-1} = d$$

Example (8)

The sum to n terms of a series is $S_n = 8n^2 - 4n$. Calculate the n^{th} term. Hence prove that the sequence is an arithmetic progression.

$$\begin{aligned} T_n &= S_n - S_{n-1} \\ &= 8n^2 - 4n - [8(n-1)^2 - 4(n-1)] \\ &\vdots \\ &= 16n - 12 \end{aligned}$$

$$\begin{aligned} T_n - T_{n-1} &= 16n - 12 - [16(n-1) - 12] \\ &= 16n - 12 - 16n + 16 + 12 \\ &= 16 \end{aligned}$$

Since $T_n - T_{n-1}$ is a constant, the sequence is an A.P.

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