APGP Tutorial 2

1. [2010/MI/I/3]

   The fourth, ninth and nineteenth term of an arithmetic progression are consecutive terms of a geometric progression.

   (a) Show that the common ratio of the geometric progression is 2. [3]

   (b) The twentieth term of the arithmetic progression is 63. Find its first term and common difference. [3]

   (c) The sum of the first n terms of the arithmetic progression is denoted by S_n. Using the results in (b), find the least value of n for which S_n exceeds 200. [3]

   [(b)$a = 6, d = 3$, (c)$n = 11$]

2. [2009/IJC/I/12]

   A geometric series G is given by

   $$e - e^{1+\alpha} + e^{1+2\alpha} - e^{1+3\alpha} + \dots,$$

   where α is a negative real number.

   (a) Show that G is convergent.

   (b) The sum to infinty of G is S. Given that the sum to infinity of the even numbered terms of G, i.e. the second, fourth, sixth, ... terms, is $-\frac{1}{3}S$, find the exact value of α.

   [(b) -ln 4]

3. [2018/NYJC/II/1]

   Sarah carried out a series of experiments which involved using decreasing amounts of a chemical. In the first experiment, she used 4 grams of the chemical and the amount of chemical used formed a geometric progression. In the 25th experiment, she used 1 gram of the chemical.

   i. Find the total amount of chemical she used in the first 25 experiments. [4]

   ii. Show that the theoretical maximum total amount of chemical she would use will not exceed 71.3 grams. [1]

   Robert carried out the same series of experiments. He also used decreasing amounts of the same chemical but the amount of chemical used formed an arithmetic progression with common difference d. If the total amount of chemical that both Sarah and Robert used for the first 25 experiments were the same, and the amount of chemical Robert used for the 25th experiment was still 1 gram, find the value of d and the amount of chemical he used for the first experiment. [4]

   [(i) 54.5 (iii)$d \approx -0.0982$, $a \approx 3.36$]

4. [2010/ACJC/I/10a]

   The first 2 terms of a geometric progression are a and b ($b < a$). If the sum of the first n terms is equal to twice the sum to infinity of the remaining terms, prove that $a^n = 3b^n$. [3]

5. [2010/RI/I/14a]

   The sum of the first n terms of a series, S_n, is given by $\frac{p^n}{5^{n-1}} - 5$, where p is a non-zero constant and $p \neq 5$.
   Obtain an expression for T_n, the n^{th} term of the series and prove that this is a geometric series.
   Find the range of values of p for the sum to infinity to exist. [5]

   [$T_n = \frac{p^{n-1}}{5^{n-2}}\left(\frac{p}{5} - 1\right)$; $-5 < p < 5$]

6. [2009/NYJC/I/9]

   (a) In a convergent geometric progression, the sum of the first n terms is equal to the sum of the remaining terms. Given also that the $(n+1)^{th}$ term is $\frac{1}{2}$, determine the value of the $(2n+1)^{th}$ term.

   (b) An arithmetic progression has first term a and common difference d. The eight, third and second term of the progression are successive terms of an infinite geometric progression.

   If the first term of the geometric progression is 10, find the sum of the even-numbered terms of the progression.

   A sequence is formed in which the n^{th} term is given by $\ln|u_n u_{n+1}|$, where u_n is the n^{th} term of the geometric progression. Show that the sequence forms an arithmetic progression and state the value of the common difference.

   [(a)$\frac{1}{4}$ (b) $\frac{25}{12}$; $-2\ln 5$].

7. [2010/TPJC/I/3]

   An increasing arithmetic progression has first term a and common difference d. The nth term denoted by T_n is such that T_1, T_4 and T_8 are in geometric progression, show that $d = \frac{a}{9}$. Given that $T_{20} + T_{22} + T_{24} + \cdots + T_{50} = 1376$. Find also the first term of the arithmetic progression. [6]

   [$a = 18$]

8. [2011/AJC/I/12]

   The sum of the first n terms of a series, S_n, is given by $S_n = \frac{1}{a}\left[1-(a-1)^n\right]$, where a is a constant and $a \neq 1$, $n \in \mathbb{Z}^+$. Obtain a expression for the n^{th} term of the series, T_n and prove that S_n is a geometric series. [3]

   If the sequence $\{T_n\}$ is now grouped as follows: $(T_1), (T_2, T_3, T_4), (T_5, T_6, T_7, T_8, T_9)$, ...where each subsequent bracket has 2 terms more than the previous bracket, find

   (a) the total number of terms in the first n brackets. [2]

   (b) the middle term of the 11^{th} bracket in terms of a. [2]

   (c) the range of values of a for the sum to infinity of the series to exist. Hence, find the least value of n for the sum of all the terms in the first n brackets to be within 0.1% of the sum to infinity of the series when $a = \frac{39}{20}$. [4]

   [$T_n = \frac{(a-1)^{n-1}}{a}(2-a)$; (a)n^2, (b)$T_{111} = \frac{1}{a}(a-1)^{110}(2-a)$] (c) $0 < a < 2$; $n = 12$]