Complex Numbers Tutorial 1

1. [2018/HCI/Prelim/I/9a]

   Showing your working clearly, find the complex numbers z and w which satisfy the simultaneous equations

   $$4iz - w = 9i - 13,$$
   $$(4 + 2i)w^* = z + 3i.$$

   [4]

   $[w = 1 - i,\ z = 2 + 3i]$

2. [2010/YJC/Prelim/I/9a]

   The complex numbers z and w are such that $z = 1 + ip, w = 1 + iq$, where p and q are real and p is positive. Given that $zw = 3 - 4i$, find the exact values of p and q. [4]

   $[\ p = -2 + \sqrt{6};\ q = -2 - \sqrt{6}\ ]$

3. [2010/SAJC/Prelim/I/4]

   Solve the equation $w^2 = 3 + 4i$, expressing your answer(s) in the form $x + iy$. [3]

   $[w = \pm(2 + i)\ ]$

4. [2010/TPJC/Prelim/I/13a]

   i. Determine the complex numbers u and v for which $z^2 + (6 - 2i)z = (z - u)^2 - v$, for all $z \in \mathbb{C}$. [2]

   ii. Find the square roots of $7 - 24i$. Hence, solve the quadratic equation $z^2 + (6 - 2i)z = -1 - 18i$. [4]

   [(i) $u = -3 + i$ and $v = 8 - 6i$ (ii) $4 - 3i$ or $-4 + 3i$; $z = -7 + 4i$ or $z = 1 - 2i$]

5. [2010/NJC/Prelim/I/10a]

   Given that $2 + 3i$ is a solution to the equation

   $$z^2 + (a - i)z^* + 16 + bi = 0,$$

   where z^* is the conjugate of the complex number z, find the values of a and b, where a and b are real constants. [3]

   $[a = -4, b = -22]$

6. [2010/NYJC/Prelim/I/2a]

   The complex numbers z and w are $1 + ai$ and $b - 2i$ respectively where a and b are real and a is negative. Given that $zw^* = 8i$, find the exact values of a and b. [3]

   $[a = -\sqrt{3}, b = -2\sqrt{3}]$

7. [2014/SAJC/Prelim/II/4a]

   Find the complex number z in the form $x + yi$ where $x, y \in \mathbb{R}$ such that

   $$\frac{iz}{z - 2z^* - 2} = -1.$$

   [3]

   $[z = -3 + i]$

8. [2009/ACJC/Prelim/I/10]

   Without the use of a calculator, find the complex numbers z and w in the form $a + ib$ that satisfy the two simultaneous equations

   $$z + (2 + i)w = -9 + 16i \quad \text{and} \quad z^* + w = 3i.$$

[4]

$[z = 2 + 4i, w = -2 + 7i\ ]$

9. [2010/MI/Prelim/I/10a]

   [You are not allowed to use the GC in this question.]

   Given that the equation $z^4 - z^3 - 9z^2 + 29z - 60 = 0$ has a root of the form $z = 1 + ki$, where k is a non-zero real number, find the possible values of k. [3]

   $[k = \pm 2]$

10. [2010/SRJC/Prelim/I/7ai]

    A complex number w is such that $ww^* + 64\sqrt{3}i + 16iw = 0$ and $\text{Im}(w) < 5$, where w^* is the conjugate of w.

    Find w in the form $x + iy$, where $x, y \in \mathbb{R}$. [3]

    $[w = -4\sqrt{3} + 4i]$

