Differential Equation Tutorial 1

1. [2010/IJC/Prelim/I/6]

   (a) Show, by means of substitution $w = x^2y$, that the differential equation

   $$x\frac{dy}{dx} + 2y + 3xy = 0$$

   can be reduced to the form

   $$\frac{dw}{dx} = -3w$$

   [2]

   (b) Hence find y in terms of x, given that $y = -\frac{1}{2}$ when $x = 2$. [4]

   [(b) $y = -\frac{2}{x^2}e^{6-3x}$]

2. [2010/NJC/Prelim/I/6]

   (a) By using the substitution $y = vx$, find the general solution of the differential equation

   $$x\frac{dy}{dx} = 3x + y - 2$$

   [4]

   (b) State the equation of the locus where the stationary points of the solution curves lie. [1]

   [$y = 3x \ln|x| + 2 + Cx$ (a) $y = -3x + 2$]

3. [2010/JJC/Prelim/II/2]

   (a) A particular solution of a differential equation is given by $(x + y)^2 = 2xy - \frac{2}{3}y^3$. Show that

   $$(y^2 + y)\frac{dy}{dx} = -x$$

   [2]

   (b) A second, related, family or curves is given by the differential equation

   $$x\frac{dy}{dx} = y^2 + y$$

   By means of the substitution $y = ux$, show that the general solution for y, in terms of x, is

   $$y = \frac{-x}{x + c},$$

   where c is an arbitrary constant. [3]

4. [2010/CJC/Prelim/II/4a,b]

   (a) Verify that $y = x$ is a particular solution of the differential equation

   $$\frac{dy}{dx} = \frac{x^2 + y^2}{2xy}, \ x, y \neq 0$$

   [2]

   (b) Show that the substitution $y = ux$ reduces the differential equation

   $$\frac{dy}{dx} = \frac{x^2 + y^2}{2xy}$$

   to the differential equation

   $$x\frac{du}{dx} = \frac{1 - u^2}{2u}.$$

Hence find the general solution of the differential equation

$$\frac{dy}{dx} = \frac{x^2 + y^2}{2xy}.$$

[3]

(c) i. Due to a rapid disease outbreak, the population of fish in a river, x (in thousands), is believed to obey the differential equation

$$\frac{d^2x}{dt^2} = 4ae^{-2t}$$

where t is the time in days, and $a > 0$ is a constant. Given that the entire population of fish is wiped out by the disease eventually, show that the general solution of the differential equation is $x = ae^{-2t}$. [3]

ii. Explain the meaning of a, in the context of the question. Sketch the solution curves of the differential equation for $a = 1$ and 2. [2]

[b $y^2 = x^2 - Ax$ (c)(ii) a represents the initial population of the fish (in thousands)]

5. [2017/AJC/I/2]

Show that the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} + \frac{3xy}{1 - 3x^2} - x + 1 = 0$$

may be reduced by means of the substitution $y = u\sqrt{1 - 3x^2}$ to

$$\frac{\mathrm{d}u}{\mathrm{d}x} = \frac{x - 1}{\sqrt{1 - 3x^2}}$$

Hence find the general solution for y in terms of x. [5]

[$y = -\frac{1}{3}(1 - 3x^2) - \frac{\sqrt{1-3x^2}}{\sqrt{3}} \sin^{-1}(\sqrt{3}x) + C\sqrt{1 - 3x^2}$]

6. [2017/PJC/II/2]

By differentiating $\cos x \frac{\mathrm{d}y}{\mathrm{d}x}$ with respect to x, solve the differential equation

$$\cos x \frac{\mathrm{d}^2y}{\mathrm{d}x^2} - \sin x \frac{\mathrm{d}y}{\mathrm{d}x} = \sec^2 x + \cos 2x,$$

giving y in terms of x. [6]

[$y = \sec x - \cos x + C \ln|\sec x + \tan x| + D$]

7. [2018/DHS/I/6a]

(a) By using the substitution $y = zx^2$, find the general solution of the differential equation

$$x^2 \frac{\mathrm{d}y}{\mathrm{d}x} = 2xy - y^2, \quad \text{where } x \neq 0.$$

[4]

i. Sketch the solution curve that passes through (2, -4), indicating any stationary points and asymptotes clearly. [4]

ii. State the particular solution for which y has no turning point. [1]

(b) A differential equation is of the form $\frac{\mathrm{d}y}{\mathrm{d}x} + y = px + q$, where p and q are constants. Its general solution is $y = 4x - 1 + De^{-x}$, where D is an arbitrary constant. Find the values of p and q. [2]

[(a)$y = \frac{x^2}{x-C}$ (b)$p = 4, q = 3$]

8. [2012/AJC/Prelim/II/4a]

   By using the substitution $u = x - y$, solve the differential equation

$$\frac{dy}{dx} + 4(x-y)^2 \cos^2 x = \sin^2 x.$$

[$\tan^{-1}[2(x-y)] = x + \frac{1}{2}\sin 2x + c$]