# Inequalities Tutorial 1

1. [2012/MI/I/2]

   (a) Without using a calculator, solve the inequality $\frac{2x^2+2x+1}{x^2-2x+1} \geq 1$. [4]

   (b) Hence, solve $\frac{2(\ln x)^2+\ln x^2+1}{(\ln x)^2-\ln x^2+1} \geq 1$. [3]

   [(a) $x \leq -4$ or $x \geq 0$, $x \neq 1$ (b) $x \leq e^{-4}$ or $x \geq 1$ $x \neq e$]

2. [2010/HCI/Prelim/I/1]

   (a) Solve the inequality

   $$\frac{x}{x-2} \leq \frac{4}{(x-2)^2}$$

   giving your answer in exact form. [3]

   (b) Hence solve $\frac{e^x}{e^x+2} \leq \frac{4}{(e^x+2)^2}$. [2]

   [$1-\sqrt{5} \leq x \leq 1+\sqrt{5}$, $x \neq 2$ (a) $x \leq \ln(\sqrt{5}-1)$]

3. [2015/RI/Prelim/I/5]

   Without using a calculator,

   i. Solve the inequality $\frac{8-5x}{2x+1} \leq 2-x$. [4]

   ii. hence find the set of values of $\theta$ for which $\frac{8-5\sin\theta}{2\sin\theta+1} \leq 2-\sin\theta$, where $0 < \theta < 2\pi$. [3]

   [(i)$x < -\frac{1}{2}$ or $1 \leq x \leq 3$ (ii) $\{\theta \in \mathbb{R} | \frac{7\pi}{6} < \theta < \frac{11\pi}{6}$ or $\theta = \frac{\pi}{2}\}$]

4. [2009/SAJC/Prelims/I/2]

   (a) Using an algebraic method, solve the inequality $\frac{2x+11}{x^2+2} > 4$, leaving your answer in exact form. [3]

   (b) Hence by completing the square or otherwise, solve the inequality

   $$\frac{2x+7}{x^2-4x+6} > 4.$$

   [3]

   [(a) $\frac{1-\sqrt{13}}{4} < x < \frac{1+\sqrt{13}}{4}$ (b) $\frac{9-\sqrt{13}}{4} < x < \frac{9+\sqrt{13}}{4}$]

5. [2010/TPJC/Prelim/I/4]

   Given that $x$ is real, prove that $4x^2+2x+1$ is always positive.

   (a) Hence, without using a calculator, solve the inequality $2x + \frac{1}{1+2x} > 0$. [4]

   (b) Deduce the range of values of $\frac{x+2}{x} + \frac{x}{x+2} > 1$. [3]

   [(a) $x > -\frac{1}{2}$ (b) $x < -2$ or $x > 0$]

6. [2012/PJC/I/4]

   Without the use of the graphic calculator, solve the inequality

   $$\frac{4x^2+4x+1}{x^2+x+1} > 0.$$

   Hence, solve the inequality $\frac{x^2+4x+4}{x^2+x+1} > 0$. [5]

   [$x \in \mathbb{R}, x \neq -\frac{1}{2}$; $x \in \mathbb{R}, x \neq -2$]

   *Its not the end! Flip for more questions :)

7. [2015/JJC/Prelim/I/1]

   Solve $\frac{2x+1}{x^2+x+1} < 1$ without the aid of a graphic calculator. Hence solve $\frac{3-2x}{x^2-3x+3} < 1$. [5]

   $[x < 0 \text{ or } x > 1;\ x > 1 \text{ or } x < 0]$

8. [2015/AJC/Prelim/I/1]

   Without using a graphic calculator, solve the inequality $\frac{x+2}{2x-1} < 2x + 1$. [3]

   Hence find the exact solution of the inequality $\frac{2x^2+1}{2-x^2} < \frac{2+x^2}{x^2}$. [3]

   $[-\frac{3}{4} < x < \frac{1}{2} \text{ or } x > 1; \qquad x > \sqrt{2} \text{ or } x < -\sqrt{2} \text{ or } -1 < x < 1, x \neq 0\ ]$