Complex Numbers Tutorial 4

1. [2011/RI/Prelim/II/1]

   (a) Given that one of the roots of the equation $z^4 - az^3 + 10z - 25 = 0$ is $1 + 2i$ where a is real, show that $a = 2$. Without using the graphic calculator, find the other roots of the equation in exact form. [4]

   (b) Hence find the roots of the equation $(w-1)^4 + 2(w-1)^3 - 10(w-1) - 25 = 0$, giving your answers in exact form. [2]

   [(a) $z = 1+2i, 1-2i, \sqrt{5}, -\sqrt{5}$ (b) $w = 2i, -2i, 1-\sqrt{5}, 1+\sqrt{5}$]

2. [2013/YJC/Prelim/II/4]

   i. Let $f(z) = z^3 - z^2 + bz + 4, b \in \mathbb{R}$. Given $f(1 + i\sqrt{3}) = 0$, obtain all roots of $f(z) = 0$. [3]

   ii. Hence find the exact roots of the equation $4w^3 + bw^2 - w + 1 = 0$. [2]

   [(i)$1 - i\sqrt{3}$, -1 (ii) $-1, \frac{1-i\sqrt{3}}{4}, \frac{1+i\sqrt{3}}{4}$]

3. [2012/RVHS/Prelim/I/1]

   i. One root of the equation $z^4 - 2z^3 + 14z^2 + az + b = 0$, where a and b are real, is $z = 1 + 2i$. Find the values of a and b and the other roots. [5]

   ii. Deduce the roots of the equation $z^4 + 2iz^3 - 14z^2 - 18iz + 45 = 0$. [2]

   [(i) $a = -18, b = 45$; $z = 1+2i, 1-2i, 3i, -3i$ (ii) $z = 2-i, -2-i, 3, -3$]

4. [2010/VJC/Prelim/I/5b]

   Given that $1 + i\sqrt{2}$ is a root of the equation

   $$3z^3 + az^2 + bz + 3 = 0.$$

   Find the values of the real numbers a and b. [3]

   [$a = -5, b = 7$]

5. [2015/HCI/Prelim/I/10a]

   The equation $z^3 - az^2 + 2az - 4i = 0$, where a is a constant, has a root i.

   i. Briefly explain why i^* may not necessarily be a root of the equation. [1]

   ii. Show that $a = 2 + i$. [2]

   iii. Hence, find the remaining roots of the equation in exact form. [5]

   [(iii) $1+\sqrt{3}i, 1-\sqrt{3}i$]

6. [2015/RI/Prelim/I/11b]

   The equation $z^3 + az^2 + 6z + 2 = 0$, where a is a real constant, has 2 roots which are purely imaginary. Find the value of a and solve the equation. [4]

   [$\sqrt{6}i, -\sqrt{6}i$ and $-\frac{1}{3}$]

7. [2015/DHS/Prelim/I/1]

   A graphic calculator is not to be used in answering this question

   i. Find the value of $(1 + 4i)^2$, showing clearly how obtain your answer. [1]

   ii. Given that $1 + 2i$ is a root of the equation

   $$z^2 - z + (a + bi)$$

   find the values of the real numbers a and b. [2]

iii. For these values of a and b, solve the equation in part (ii). [2]

[(i) $-15+8i$ (ii) $a=4$, $b=-2$ (iii) $1+2i$, $-2i$]

8. [2017/NYJC/I/3]

Do not use a calculator in answering this question.

i. Explain why the equation $z^3+az^2+az+7=0$ cannot have more than two non-real roots, where a is a real constant. [1]

ii. Given that $z=-7$ is a root of the equation in (i), find the other roots, leaving your answers in the form $\mathrm{r}\mathrm{e}^{\mathrm{i}\theta}$, where $r>0$ and $-\pi<\theta\leq\pi$. [4]

iii. Hence, solve the equation $\mathrm{i}z^3+8z^2-8\mathrm{i}z-7=0$, leaving your answers in the form $\mathrm{r}\mathrm{e}^{\mathrm{i}\theta}$, where $r>0$ and $-\pi<\theta\leq\theta$. [2]

[(i) $z=7e^{\mathrm{i}\pi}, e^{\frac{-\mathrm{i}2\pi}{3}}, e^{\frac{\mathrm{i}2\pi}{3}}$ (ii) $z=7e^{\frac{\mathrm{i}\pi}{2}}, e^{\frac{\mathrm{i}\pi}{6}}, e^{\frac{\mathrm{i}5\pi}{6}}$]