Differential Equation

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Separable Variables

$$\frac{dy}{dx} = f(y)g(x) \qquad \Rightarrow \qquad \frac{1}{f(y)}\frac{dy}{dx} = g(x)$$

$$\Rightarrow \qquad \int \frac{1}{f(y)}\ dy = \int g(x)\ dx$$

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Example (1)

Find the general solution to the differential equation $\frac{dy}{dx} = \frac{2y-1}{x+2}$.

Example (2)

Find the particular solution of the differential equation $y\frac{dy}{dx} + e^{-3y^2} = 0$, given that $y = 0$ when $x = 0$. $[-\frac{1}{6}e^{3y^2} = x - \frac{1}{6}]$

Example (3)

Find the general solution to the differential equation $x\frac{dy}{dx} = 1 + 2y$.

$$x\frac{dy}{dx} = 1 + 2y$$

$$\int \frac{1}{1+2y}\ dy = \int \frac{1}{x}\ dx$$

$$\frac{1}{2}\int \frac{2}{1+2y}\ dy = \ln|x| + C$$

$$\frac{1}{2}\ln|1+2y| = \ln|x| + C$$

$$\ln|1+2y| = 2\ln|x| + 2C$$

$$\ln|1+2y| = \ln x^2 + 2C$$

$$|1+2y| = e^{\ln x^2 + 2C}$$

$$|1+2y| = e^{\ln x^2} \cdot e^{2C}$$

$$1+2y = \pm e^{\ln x^2} \cdot e^{2C}$$

$$1+2y = \pm e^{2C} \cdot e^{\ln x^2}$$

$$1+2y = Ae^{\ln x^2}, \quad \text{where } A = \pm e^{2C}$$

$$\therefore y = \frac{Ax^2 - 1}{2}$$

Example (3)

Find the general solution to the differential equation $x\frac{dy}{dx} = 1 + 2y$.

$$
\begin{aligned}
x\frac{dy}{dx} &= 1 + 2y \\
\int \frac{1}{1+2y}\,dy &= \int \frac{1}{x}\,dx \\
\frac{1}{2}\int \frac{2}{1+2y}\,dy &= \ln|x| + C \\
\frac{1}{2}\ln|1+2y| &= \ln|x| + C \\
\ln|1+2y| &= 2\ln|x| + 2C \\
\ln|1+2y| &= \ln x^2 + 2C \\
|1+2y| &= e^{\ln x^2 + 2C} \\
|1+2y| &= e^{\ln x^2} \cdot e^{2C} \\
1+2y &= \pm e^{\ln x^2} \cdot e^{2C} \\
1+2y &= \pm e^{2C} \cdot e^{\ln x^2} \\
1+2y &= Ae^{\ln x^2}, \quad \text{where } A = \pm e^{2C} \\
\therefore y &= \frac{Ax^2 - 1}{2}
\end{aligned}
$$

By Substitution

Step 1. Differentiate the given substitution equation wrt x.

Step 2. Replace $\frac{dy}{dx}$ and y accordingly with the expression in step 1.

Step 3. Solve the differential equation accordingly.

Step 4. Replace the variables to the original.

Example (4)

Using the substitution $v = y - x$, solve $\frac{dy}{dx} = \frac{1+y-x}{5+y-x}$.

Step 1. Differentiate the substitution.

$$v = y - x$$
$$\frac{dv}{dx} = \frac{dy}{dx} - 1$$
$$\frac{dy}{dx} = \frac{dv}{dx} + 1$$

Step 2. Replacement of variables.

$$\frac{dy}{dx} = \frac{1 + y - x}{5 + y - x}$$
$$\frac{dv}{dx} + 1 = \frac{1 + (v + x) - x}{5 + (v + x) - x}$$
$$\frac{dv}{dx} + 1 = \frac{1 + v}{5 + v}$$
$$\frac{dv}{dx} = \frac{1 + v}{5 + v} - 1$$
$$= \frac{-4}{5 + v}$$

Example (4)

Using the substitution $v = y - x$, solve $\frac{dy}{dx} = \frac{1+y-x}{5+y-x}$.

Step 1. Differentiate the substitution.

$$
\begin{aligned}
v &= y - x \\
\frac{dv}{dx} &= \frac{dy}{dx} - 1 \\
\frac{dy}{dx} &= \frac{dv}{dx} + 1
\end{aligned}
$$

Step 2. Replacement of variables.

$$
\begin{aligned}
\frac{dy}{dx} &= \frac{1+y-x}{5+y-x} \\
\frac{dv}{dx} + 1 &= \frac{1+(v+x)-x}{5+(v+x)-x} \\
\frac{dv}{dx} + 1 &= \frac{1+v}{5+v} \\
\frac{dv}{dx} &= \frac{1+v}{5+v} - 1 \\
&= \frac{-4}{5+v}
\end{aligned}
$$

Example (4)

Using the substitution $v = y - x$, solve $\frac{dy}{dx} = \frac{1+y-x}{5+y-x}$.

Step 3. Solve the DE.

$$\frac{dv}{dx} = \frac{-4}{5+v}$$

$$\frac{5+v}{-4}\frac{dv}{dx} = 1$$

$$\int \frac{5+v}{-4}\, dv = \int 1\ dx$$

$$-\frac{1}{4}\left(5v + \frac{v^2}{2}\right) = x + C$$

Step 4. Replacement of variables.

$$-\frac{1}{4}\left(5(y-x) + \frac{(y-x)^2}{2}\right) = x + C$$

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Example (5)

Find the general solution of the following differential equation by means of the suggested substitution:

$$x\frac{dy}{dx} - y = x(x - y); \quad y = vx$$

Differentiate $y = vx$ w.r.t. x,

$$\frac{dy}{dx} = v + x\frac{dv}{dx}$$

Substitute y and $\frac{dy}{dx}$ into the DE:

$$\begin{aligned}
x\left(v + x\frac{dv}{dx}\right) - vx &= x(x - vx) \\
xv + x^2\frac{dv}{dx} - vx &= x^2 - vx^2 \\
\frac{dv}{dx} &= 1 - v \\
\int \frac{1}{1-v}\ dv &= \int 1\ dx \\
-\int \frac{-1}{1-v}\ dv &= \int 1\ dx \\
-\ln|1-v| &= x + C \\
\ln|1-v| &= -x - C \\
|1-v| &= e^{-x}\cdot e^{-C} \\
1 - v &= \pm e^{-C}\cdot e^{-x} \\
1 - v &= Ae^{-x}, \quad \text{where } A = \pm e^{-C}. \\
v &= 1 - Ae^{-x} \\
\frac{y}{x} &= 1 - Ae^{-x} \\
y &= x - Axe^{-x}
\end{aligned}$$

Example (6)

Find the general solution of the differential equation $\frac{d^2y}{dx^2} = ae^{-2x}$, where a is a constant.

Solution:

$$\frac{d^2y}{dx^2} = ae^{-2x}$$

$$\int \frac{d^2y}{dx^2}\, dx = \int ae^{-2x}\, dx$$

$$\frac{dy}{dx} = \frac{ae^{-2x}}{-2} + C$$

$$\int \frac{dy}{dx}\, dx = \int \left(\frac{ae^{-2x}}{-2} + C\right)\, dx$$

$$y = \frac{ae^{-2x}}{4} + Cx + d$$

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[2011/RI/Prelim/I/8]

(a) Given that y is a function of x, find $\frac{d}{dx}(ye^{3x})$ in terms of x, y and $\frac{dy}{dx}$. [1]

(b) Hence, or otherwise, find the general solution of $\frac{dy}{dx} + 3y = 1$, expressing y in terms of x. [2]

(a) $\frac{d}{dx}(ye^{3x}) = y(3e^{3x}) + e^{3x}\frac{dy}{dx} = 3ye^{3x} + \frac{dy}{dx}e^{3x}$

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$$\int \frac{d}{dx}(ye^{3x})\, dx = \int e^{3x}\, dx$$

$$ye^{3x} = \frac{e^{3x}}{3} + C$$

$$y = \frac{1}{3} + Ce^{-3x}, \quad \text{where } C \text{ is an arbitrary constant.}$$

Example (7)

[2011/RI/Prelim/I/8]

(a) Given that y is a function of x, find $\frac{d}{dx}(ye^{3x})$ in terms of x, y and $\frac{dy}{dx}$. [1]

(b) Hence, or otherwise, find the general solution of $\frac{dy}{dx} + 3y = 1$, expressing y in terms of x. [2]

(a) $\frac{d}{dx}(ye^{3x}) = y(3e^{3x}) + e^{3x}\frac{dy}{dx} = 3ye^{3x} + \frac{dy}{dx}e^{3x}$

(b) $\frac{dy}{dx} + 3y = 1$, multiplying both sides by e^{3x},

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