# Permutation and Combination Tutorial 1: Solutions

1. Choose 1 men to be in between: $^3C_1$
   Permute the 2 sisters: 2!
   Treat this man and the 2 sisters as a group, permute together with the remaining 3 people: 4!

   Total no. of ways $= {}^3C_1 \times 2! \times 4! = 144$

2. (i) No. of ways $= 8! = 40320$

   (ii) No. of ways to arrange remaining 6 people: 6!
   No. of ways to slot in the secretary and manager: $^7C_2$
   No. of ways to permute the 2 of them: 2!

   Total no. of ways $= 6! \times^7 C_2 \times 2! = 30240$

3. (i) No. of ways $= 9! = 362,880$

   (ii) No. of ways to arrange remaining 6 people: 6!
   No. of ways to slot in Tan family: $^7C_3$
   No. of ways to permute the 3 of them: 3!

   Total no. of ways $= 6! \times^7 C_3 \times 3! = 151,200$

4. (a) No. of suits: 4
   No. of ways to choose 5 cards from one suit: $^{13}C_5$

   Total no. of ways $= 4 \times^{13} C_5 = 5148$

   (b) Econs and physics: $5 \times 3 = 15$
   Physics and Art: $3 \times 2 = 6$
   Art and Econs: $2 \times 5 = 10$

   Total no. of ways $= 15 + 6 + 10 = 31$

   (c) $^8C_4 \times^4 C_2 = 420$

5. i) $6! = 720$

   ii) $2^6 = 64$

   iii) $2^6 - 1 = 63$

6. **Case 1: 2 tall 0 short**
   No. of ways $= {}^3C_2$
   No. of ways to choose remaining 6 players from $8 = {}^8C_6$
   Total no. of ways $= {}^3C_2 \times^8 C_6 = 84$

   **Case 2: 2 tall 1 short**
   No. of ways $= {}^3C_2 \times^2 C_1$
   No. of ways to choose remaining 5 players from $8 = {}^8C_5$
   Total no. of ways $= {}^3C_2 \times^2 C_1 \times^8 C_5 = 336$

   **Case 3: 3 tall 0 short**
   No. of ways $= {}^3C_3$
   No. of ways to choose remaining 5 players from $8 = {}^8C_5$
   Total no. of ways $= {}^3C_3 \times^8 C_5 = 56$

   **Case 4: 3 tall 1 short**
   No. of ways $= {}^3C_3 \times^2 C_1$
   No. of ways to choose remaining 4 players from $8 = {}^8C_4$
   Total no. of ways $= {}^3C_3 \times^2 C_1 \times^8 C_4 = 140$

   Total no. of ways $= 616$

7. (a) Choose 2 members from CI to perform DI: ${}^{9}C_2$
Choose 2 members from the remaining 29 people to perform FR: ${}^{29}C_2$
Choose 2 members from the remaining 27 people to perform recital:${}^{27}C_2$
Total no. of ways $= {}^{9}C_2 \times^{29} C_2 \times^{27} C_2 = 5130216$

(b) **Strategy:** Take total number of duty groupings with no restrictions - number of groupings where Jill is NOT on duty

**Number of groupings where Jill is NOT on duty:**
Choose 2 members from CI(excluding Jill) to perform DI: ${}^{8}C_2$
Choose 2 members from the remaining 28 people to perform FR: ${}^{28}C_2$
Choose 2 members from the remaining 26 people to perform recital:${}^{26}C_2$
No. of groupings where Jill is NOT on duty: ${}^{8}C_2 \times^{28} C_2 \times^{26} C_2$

No of ways Jill is on duty = 5130216 - ${}^{8}C_2 \times^{28} C_2 \times^{26} C_2 = 1690416$

(c) **Strategy:** Take total number of duty groupings with no restrictions - number of groupings where Jack is NOT on duty

**Number of groupings where Jack is NOT on duty:**
Choose 2 members from CI to perform DI: ${}^{9}C_2$
Choose 2 members from the remaining 28 people to perform FR: ${}^{28}C_2$
Choose 2 members from the remaining 26 people to perform recital:${}^{26}C_2$
No. of groupings where Jack is NOT on duty: ${}^{9}C_2 \times^{28} C_2 \times^{26} C_2$

No of ways = 5130216 - ${}^{9}C_2 \times^{28} C_2 \times^{26} C_2 = 707616$

8. (a) ${}^{20}C_7 = 77520$

(b) Think of the problem as distributing gifts to children.
For each gift, there are 20 choices (20 children).

No. of ways $= 20 \times 20 \times 20 \times 20 = 20^4 = 160,000$

(c) $9! = 362880$

(d) i. Of the remaining 5 seats, $B, C, F, G, H$, we choose 3 seats for them and permute: ${}^{5}C_3 \times 3!$
Permute the rest of the 6 people: 6!

No. of ways $= {}^{5}C_3 \times 3! \times 6! = 43200$

ii. **Strategy:** Find the number of ways which the people can seat without restriction (part c), subtract the number of ways which the 2 particular people are seated next to each other on the same side of the table.

**Case 1:** The 2 of them are seated on the top half of the table.

Choose 2 seats side by side: ${}^{3}C_1$ (AB, BC or CD)
Permute the 2 of them: 2!
Permute remaining 7 people: 7!
**No. of ways** $= {}^{3}C_1 \times 2! \times 7! = 30240$

**Case 2:** The 2 of them are seated on the bottom half of the table.

Choose 2 seats side by side: ${}^{4}C_1$ (EF, FG, GH or HI)
Permute the 2 of them: 2!
Permute remaining 7 people: 7!
**No. of ways** $= {}^{4}C_1 \times 2! \times 7! = 40320$

**Required no. of ways** $= 362880 - 30240 - 40320 = 292320$

 

9. Case 1: 4 and 5 in the second and fourth positions

   $2! \times 3! = 12$

   Case 2: 3 and 5 in the second and fourth positions

   1 3 2 5 4

   2 3 1 5 4

   4 5 1 3 2

   4 5 2 3 1

   No. of cases: 4

   Total number = 12 + 4 = 16

10. i. Each room can be painted with 6 different colours.
   No of ways $= 6^4 = 1296$

   ii. No. of ways to choose 4 colours= ${}^6C_4$
   No. of ways to permute the 4 colours = 4!

   Total no. of ways $= {}^6C_4 \times 4! = 360$

   iii. No. of ways to choose 2 colours $= {}^6C_2$

   Each room can be painted with 2 colours. No of ways to paint 4 rooms $= 2^4$
   We need to subtract away the 2 cases where the 4 rooms are of the same colour.
   No of ways to paint 4 rooms with 2 colours $= 2^4 - 2$

   Total no. of ways $= {}^6C_2 \times (2^4 - 2) = 210$

11. (a) Number of foreigners = 2 + 3 =5.
   Number of ways to arrange 6 locals = 6!
   Number of ways to slot 5 foreigners amongst the 6 locals $= {}^7C_5 \times 5!$
   Total number of ways $= 6! \times {}^7C_5 \times 5! = 1814400$.

   (b) (i) Number of ways $= {}^{11}C_4 = 330$

   (ii) Number of ways to choose without males $= {}^5C_4$
   Number of ways to choose without females $= {}^6C_4$
   Total number of ways $= {}^{11}C_4 - {}^5C_4 - {}^6C_4 = 310$

   (iii) Number of ways to choose without females $= {}^6C_4$
   Number of ways to choose without foreigners $= {}^6C_4$
   Number of ways to choose 4 local male $= {}^4C_4$

| | local | foreigner |
|---|---|---|
| Male | | |
| Female | | |

   Total number of ways
   = no. of ways w/o restriction - no. of ways w/o female - no. of ways w/o foreigner + no. of ways with local male
   $= 330 - {}^6C_4 - {}^6C_4 + {}^4C_4$
   $= 301$

12. (a) Suppose the child has no restrictions. He can try 0 rides, 1 ride, 2 rides ... 14 rides, 15 rides.
   For each ride, he has 2 options: Ride or Not

No. of ways (if no restrictions): $2^{15}$

No. of ways which he did not take any of the 15 rides: 1
No. of ways which he took 1 of the 15 rides: 15

Total no. of ways if he tries at least 2: $2^{15} - 1 - 15 = 32752$

(b) i) $9! = 362880$

ii) Choose a row to be seated: ${}^{3}C_{1}$
Choose 2 seats from the row: 2 (Either the left 2 or the right 2 since they must sit tgt)
Permute the mom and the child: $2!$
Permute the rest of the 7 people: $7!$

Total no. of ways $= 3 \times 2 \times 2! \times 7! = 60480$