# Graphing Tutorial 2: Conics

**Note:** Skip questions with * if you have not done graphing transformations in school.

1. [2010/MI/Prelim/I/4]

   The curve $C$ has equation
   $$a^2(x+1)^2 - b^2y^2 = 1$$
   where $a$ and $b$ are positive constants.

   (a) Given that the curve passes through the points $(-\frac{3}{2}, 0)$ and the equations of its asymptotes are $y = 2x + 2$ and $y = -2x - 2$, show that $a = 2$ and $b = 1$. [4]

   (b) Hence sketch $C$, stating the equations of any asymptotes and the coordinates of any points of intersection with the axes. [3]

2. [2011/ACJC/II/5]

   The curves $C_1$ and $C_2$ have equations $9y^2 = (x+k)^2 - 9$ and $\frac{x^2}{k^2} + y^2 = 1$ respectively, where $k$ is a real constant such that $k > 3$.

   (a) *Describe a sequence of transformations which transforms the graph of $x^2 - y^2 = 1$ to the graph of $C_1$. [2]

   (b) i. On the same diagram, sketch the graphs of $C_1$ and $C_2$, stating clearly the coordinates of any points of intersection with the axes and the equations of any asymptotes. [5]

   ii. Find the range of values of the positive constant $a$ such that the equation
   $$\frac{x^2}{a^2} + \frac{(x+k)^2 - 9}{9} = 1$$
   has four real roots. [2]

   [(a) Scaling parallel to the $x$-axis by factor 3, translation in the negative $x$-axis direction by $k$ units b(i) $C_1$: $(3-k,\ 0)$, $(-3-k,\ 0)$, $(0,\ \pm\sqrt{\frac{k^2}{9} - 1})$; $y = \pm\frac{x+k}{3}$; $C_2$: $(\pm k, 0)$, $(0,\ \pm 1)$ b(ii) $a > k + 3$]

3. [2011/SRJC/Prelim/I/6]

   Find the equations of the asymptotes of the hyperbola $4x^2 - 24x - 9y^2 + 36y = 36$. Hence sketch the hyperbola, stating clearly the asymptotes. [3]

   Hence find the range of values of $k$, such that the equation

   $4x^2 - 24x - 9(kx+4)^2 + 36(kx+4) - 36 = 0$ has no real solutions. [3]

   [$y = \frac{2}{3}x$ and $y = -\frac{2}{3}x + 4$, $k \le -\frac{2}{3}$]

4. [2010/HCI/Prelim/I/12]

The curves $C_1$ and $C_2$ have equations $(x-2)^2 = a^2(1-y^2)$ and $y = \frac{x^2-4}{x+1}$, where $1 < a < 2$ respectively. Describe the geometrical shape of $C_1$. [1]

(a) *State a sequence of transformations which transforms the graph of $x^2 + y^2 = 1$ to the graph of $C_1$. [3]

(b) i. Sketch $C_1$ and $C_2$ on the same diagram, stating the coordinates of any points of intersection with the axes and the equations of any asymptotes. [6]

ii. Show algebraically that the $x$-coordinates of the points of intersection of $C_1$ and $C_2$ satisfy the equation

$$(x+1)^2(x-2)^2 = a^2(x+1)^2 - a^2(x^2-4)^2$$

[2]

iii. Deduce the number of real roots of the equation in part(b). [1]

[(a) scale parallel to $x$-axis by factor $a$, translate in the positive $x$-direction by 2 units b(iii) 2]

5. [2014/RVHS/P1/Q8]

The curve $C$ has equation $y = \frac{ax^2+x-3}{(x+b)(x+3)}$ where $a$ and $b$ are constants.

i. Given that the equations of the asymptotes of $C$ are $x = 2$ and $y = 1$, determine the values of $a$ and $b$. [2]

ii. Sketch $C$, showing clearly any axial intercepts, asymptotes and stationary points. [3]

iii. By sketching the graph of $(x+3)^2 + (y-1)^2 = k^2$ on the same diagram, determine the range of values of $k$ such that the equation $(x+3)^2 + \left(\frac{ax^2+x-3}{(x+b)(x+3)} - 1\right)^2 = k^2$ where $a$ and $b$ are values found in part (i), has at least one positive root. [4]

[(i) $a = 1, b = -2$ (iii) $k < -\frac{\sqrt{37}}{2}$ or $k > \frac{\sqrt{37}}{2}$]

6. [2010/TJC/Prelim/II/4b]

The curve $C$ has equation $y = \frac{-x^2-2x+5}{x-1}$.

i. *Describe how the curve with equation $y = x - 4 - \frac{2}{x}$ can be transformed into $C$. [3]

ii. State the equations of all the asymptotes of $C$ and hence sketch $C$. [2]

(You are not required to find the $x$ and $y$ intercepts.)

iii. By adding a suitable graph to the sketch in (ii), deduce the number of roots of the equation $\frac{(x-1)^2}{2^2} + (\frac{-x^2-2x+5}{x-1} + 4)^2 = 1$. [2]

[(ii) $x = 1, y = -x - 3$ (iii) 4 roots]

 