Graphing Tutorial 3: Transformation of Graphs

1. [2011/SRJC/Prelim/I/7]

   A graph with equation $y = g(x)$ undergoes in succession, the following transformations:

   A: A reflection about the x-axis

   B: A translation of 1 unit in the direction of the positive y-axis

   C: Scaling parallel to the x-axis by a factor of 3

   The equation of the resulting curve is given by $y = \frac{x-12}{2x-9}$.

   Find the equation of $y = g(x)$. [3]

   [$y = \frac{x+1}{2x-3}$]

2. [2011/SAJC/Prelim/II/1]

   The curve $y = f(x)$ undergoes, in succession, the following transformations:

   A: a translation of 4 units in the positive x-direction

   B: a reflection in the y-axis

   C: a stretch with scale factor 2 parallel to the x-axis

   The equation of the resulting curve is $y = \frac{x-1}{x+1}$, Obtain the equation of the original curve $y = f(x)$. [3]

   [$y = \frac{2x+9}{2x+7}$]

3. [2011/MI/Prelim/I/8]

   State the sequence of transformations which transform the graph $y = \frac{2a}{x+a}$ to the graph $y = \frac{6a^2}{x+a} - 2a$, where a is a positive constant. [2]

   [scaling by a factor of $3a$ units parallel to the y-axis followed by translation by $2a$ units in the negative direction of the y-axis.]

4. [2010/PJC/Prelim/I/5]

   The curve C has equation

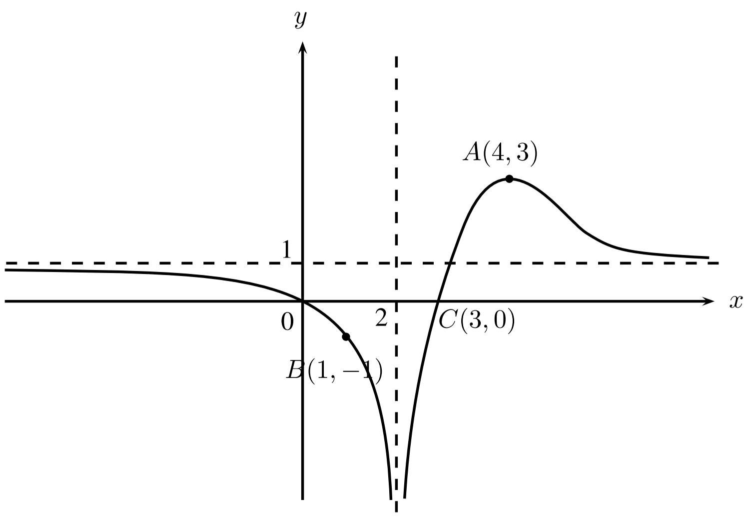
   $$y = \frac{p^2x^2 + pqx + 1}{px + q},$$

   where p and q are positive constants.

   State a sequence of transformations which transform the graph $y = x + \frac{1}{x}$ to the graph of C. [3]

   [translation of $-q$ unit in the direction of x-axis, scaling parallel to x-axis by a scale factor of $\frac{1}{p}$, translation of $-q$ units in the direction of y-axis]

5. [2010/SRJC/Prelim/II/3modified]



The diagram above shows the graph of $y = f(x)$. On separate diagrams, sketch the graphs of

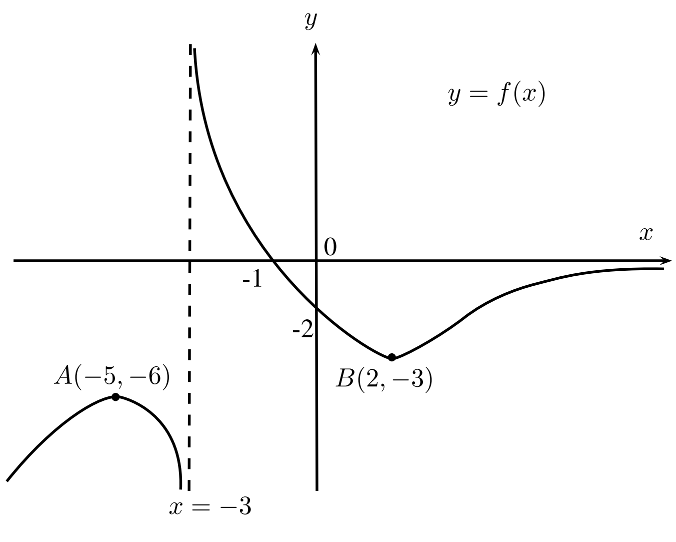
(a) $y = f(1 + 2x)$, [3]

(b) $y = 2f(x) + 1$, [3]

(c) $y = -f(-x + 2)$. [3]

showing in each case, the coordinates of the points corresponding to A, B, C and the equations of the asymptotes.
[a) $A'(1.5, 3)$ $B'(0, -1)$ $C'(1, 0)$ b) $A'(4, 7)$ $B'(1, -1)$ $C'(3, 1)$ c) $A'(-2, -3)$ $B'(1, 1)$ $C'(-1, 0)$]

6. The diagram shows the graph of $y = f(x)$, which has turning points at $A(-5, -6)$ and B$(2, -3)$. The horizontal and vertical asymptotes are $y = 0$ and $x = -3$ respectively.



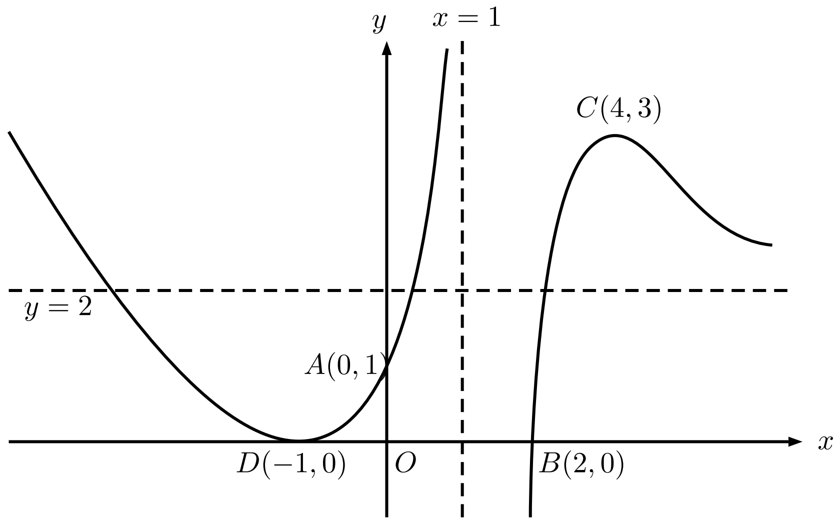
On separate diagrams, sketch the graphs of

(a) $y = -f(x) - 2$. [3]

(b) $y = |f(x + 1)|$. [3]

showing in each case, the coordinates of the points corresponding to A, B and the equations of the asymptotes. [a) $A'(-5, 4)$ $B'(2, 1)$ b) $A'(-6, 6)$ $B'(1, 3)$]

7. [2015/HCI/Prelim/I/11a]



The graph of $y = f(x)$ is shown in the diagram above. It has asymptotes $x = 1$ and $y = 2$. The points A, B, C and D have coordinates (0,1), (2,0), (4,3) and (-1,0) respectively with C and D being stationary points. Sketch the graph of

$y = f(2x - 1)$. [3]

State the coordinates of A, B, C and D whenever applicable, and the equations of any asymptotes.

[$A'(\frac{1}{2}, 1)$, $B'(\frac{3}{2}, 0)$, $C'(\frac{5}{2}, 3)$, $D'(0, 0)$]

8. [2016/JJC/Prelim/I/9]

It is given that $f(x) = x + \frac{m^2}{x-2}$, where $0 < m < 1$.

i. Sketch the graph of $y = f(x)$, showing clearly the coordinates of the turning points and the equation(s) of any asymptote(s). [5]

ii. By inserting a suitable graph to your sketch in (i), find the set of values of k, in terms of m, for which the equation $x^2 - (2 + k)x + (m^2 + 2k) = 0$ has two distinct positive roots. [4]

iii. The curve $y = f(x)$ undergoes the transformations A, B and C in succession:

A: A translation of -2 units in the direction of x-axis,

B: A stretch parallel to the x-axis with scale factor of $\frac{1}{2}$, and

C: A translation of -2 units in the direction of y-axis.

Given that the resulting curve is $y = 2x + \frac{1}{8x}$, find the value of m. [2]

[(ii) $k > 2 + 2m$ (iii) $m = \frac{1}{2}$]