Binomial Distribution Tutorial 2

1. [07/TPJC/Prelim/II/5modified]

   A factory produces chocolate which are packed into boxes of n chocolates and delivered to shops for sale. A chocolate will not meet the minimum criteria for packing for sale if it weights less than 20 grams. On average, 2% of the chocolate produced did not meet the minimum criteria.

   (a) Given that the probability that there are more than 2 chocolates in a box that do not meet minimum criteria is less than 0.03, find the largest number of n. [3]

   (b) Given that the chocolates are packed into boxes of 20,

      i. find the probability that a randonly chosen box contain at least 1 chocolate that does not meet the minimum criteria, [2]

      ii. find the probability that out of 4 randomly chosen boxes of chocolate, there are exactly 2 boxes with at least 1 chocolate that does not meet the minimum criteria. [2]

   [a) 33 b)i) 0.332 ii) 0.295]

2. [2012/SAJC/Prelim/II/13]

   On the average, 30% of the students in Calculus Madness Institute (CMI) could do the Differential Equation question in Block Test Two. The principal randomly select a class of 30 students to analyse the results.

   (a) State, in the context of this question, two assumptions needed to model the results by a binomial distribution. [2]

   (b) Find the probability that at least 6 students in that class could do that question. [2]

   (c) *Find the probability that only 2 students among the first 8 selected students in that class could do the question given that at least 6 students could do that question. [3]

   (d) The probability of no more than 5 students could do the question in a randomly selected class exceeds 0.9. Find the largest possible number of students in that class. [3]

   [(b) 0.923 (c) 0.299 (d) 11]

3. [2008/MJC/II/Q8]

   In a school with a large number of students, on average, 1 out of 5 uses AIKON brand mobile phone. In a random sample of n students, find the least value of n such that the probability that at least one student uses AIKON brand mobile phone exceeds 0.99. [least $n = 21$]

4. [2012/IJC/Prelim/II/7]

   (a) A die is biased and the probability, p, of throwing a six is known to be less than $\frac{1}{6}$. An experiment consists of recording the number of sixes in 25 throws of the die. In a large number of experiments, the standard deviation of the number of sixes is 1.5. Show that the value of p is $\frac{1}{10}$. Hence find the probability that at least 6 but fewer than 10 sixes are reocrded during a particcuar experiment. [4]

   (b) The biased die is now thrown 40 times. Find the most likely number of sixes obtained. [1]

   Note: Variance = (Standard deviation)^2 [(a) 0.0333 (b) 4]

5. [2011/DHS/Prelim/II/5]

   In a binomial probability distribution X, there are n trials and the probability of success of each trial is p. If $n = 20$ and $\mathrm{P}(X \leq 1) = 0.8$, determine the value of p. Hence find the least value of a such that $\mathrm{P}(X < a) > 0.999$. [5]

   [Ans: 6]

6. [2017/NJC/II/10]

Factory A manufactures a large batch of light bulbs. It is known that on average, 1 out of 200 bulbs manufactured by Factory A, is defective. A random sample of 180 light bulbs is inspected. The batch is accepted if the sample contains less than r defective light bulbs.

i. Explain why the context above may not be well-modelled by a binomial distribution. [1]

Assume now that the context above is well-modelled by a binomial distribution.

ii. Determine the value of r such that the probability of accepting the batch is 0.998. [2]

In Factory B, a random sample of 30 light bulbs is taken from a large batch. If the sample contains no defective light bulbs, the batch is accepted. The batch is rejected if the sample contains more than two defective light bulbs. If the sample contains one or two defective light bulbs, a second random sample of 30 light bulbs is chosen and the batch is accepted only if this second sample contains no defectives. It is known that Factory B produces $(100p)\%$ defective light bulbs.

iii. Find the probability that the batch is accepted. Leave your answer in terms of p. [3]

Forty random samples of 30 light bulbs are taken from each of the two factories A and B.

iv. Given that $p = 0.007$ and there is exactly one defective bulb, find the probability that it is from Factory B. [4]

[(ii) $r = 5$ (iii) $(1-p)^{30} + 30(1-p)^{59} + 435p^2(1-p)^{58}$ (iv) 0.584]

7. [2017/YJC/II/8]

An archaeologist examines rocks to look for fossils. On average, 10% of the rocks selected from a particular area with a large number of rocks contain fossils. The archaeologist selects a random sample of 25 rocks from this area. The number of rocks that contain fossils is denoted by X.

i. Find the probability that more than 4 but at most 10 rocks contain fossils. [2]

ii. Show that $\frac{\mathrm{P}(X=k+1)}{\mathrm{P}(X=k)} = \frac{25-k}{9(k+1)}$, for $k = 0, 1, 2, 3, \ldots, 24$. Hence, by considering $\mathrm{P}(X = k+1) > \mathrm{P}(X = k)$, find the most probable value of X. [4]

The archaeologist explores a new area. On average, p% ($p > 10$) of the rocks in the new area contain fossils. A random sample of 20 rocks from the new area is selected. Given that the probability that there are two rocks that contain fossils is 0.17, find the value of p, giving your answer correct to 2 decimal places. [3]

[(i) 0.0980 (3 s.f.) (ii) $X = 2$; $p = 18.16$ (2 d.p.)]

8. [2017/TPJC/II/6]

A geologist splits rocks to look for fossils. On average 7% of the rocks selected from a particular area contain fossils. The geologist selects a random sample of 20 rocks from this area.

i. Find the probability that at least three of the rocks contain fossils. [2]

A random sample of n rocks is selected from this area.

ii. The geologist wants to have a probability of 0.8 or greater of finding fossils in at least three of these rocks. Find the least possible value of n. [3]

In early 2017, geologists found the fossils of zilantophis schuberti, a new discovered species of winged serpent. On average, the proportion of rocks that contain fossils of zilantophis schuberti in this area is p. It is known that the modal number of fossils of zilantophis schuberti in a random sample of 10 rocks is 3.

iii. Use this information to find exactly the range of values that p can take. [4]

[(i) 0.161 (ii) 60 (iii) $\frac{3}{11} < p < \frac{4}{11}$]