Math Academy: JC Graphing Test

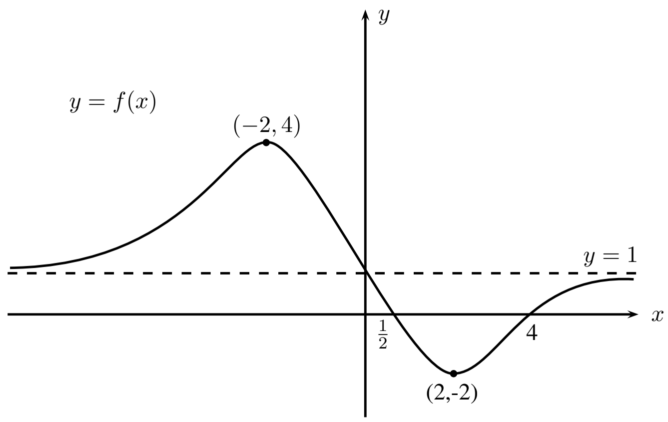
Name:

Marks: / 18

Time: 35 mins

1. State a sequence of transformations which transform the graph of $x^2 + y^2 = 1$ to the graph of $(x-1)^2 + y^2 = 4$. [3]

2.



In the diagram above, the curve $y = f(x)$ cuts the x-axis at $x = \frac{1}{2}$ and $x = 4$, has turning points at $(-2, 4)$ and $(2, -2)$, and has a horizontal asymptote $y = 1$.

Sketch, on separate diagrams, the graphs of

(a) $y = f(1-x)$, [3]

(b) $y = \frac{1}{f(x)}$, [3]

stating the equations of any asymptotes and the coordinates of any turning points and points of intersection with the axes.

3. The curve C has equation

$$y = \frac{4x^2}{x-q}$$

where q is a non-zero constant.

It is given that C has a stationary point at $x = 4$ and an asymptote $y = 4x + r$, where r is a non-zero constant.

i. Find the values of q and r. [3]

ii. Sketch C, stating clearly the equations of its asymptotes, stationary points and the coordinates of any point(s) of intersections with the axes. [3]

iii. State the set of values that y can take. [1]

iv. Using the graph in part (ii), find the range of a such that the equation

$$(x-2)^2 + \left(\frac{4x^2}{x-q} - 16\right)^2 = a$$

has a negative real root. [2]