Math Academy Vectors Test 1

Name:

Marks: / 20
Time: 36 mins

1. The equations of the three planes p_1, p_2, p_3 are

$$2x + 3y - 6z = 10,$$

$$-2x - 3y + 6z = a,$$

$$x + y + bz = 5,$$

respectively, where a, b are constants.

The planes p_1 and p_3 intersect in the line l with cartesian equation $\frac{5-x}{3} = \frac{y}{4} = z$.

i. Show that $b = -1$. [2]

ii. The point S lies on p_1 and the point R has coordinates $(-2, 4, 1)$. Given that RS is perpendicular to p_3, find the coordinates of S. [4]

The planes p_1 and p_2 are $\frac{8}{7}$ units apart.

iii. Given that $a < 0$, find the possible values of a. [4]

iv. The point P with coordinates $(5, 2, c)$ lies on p_1. Find the value of c. [1]

v. The point F is the foot of perpendicular from P to the line l. The point Q is the reflection of F in the plane p_2. Find the distance PF and hence find the area of triangle FPQ. [4]

2. i. Referred to the origin O, points A and B have position vectors $\mathbf{a}$ and $\mathbf{b}$ respectively. Point C lies on the line segment AB, such that $AC : CB = 2 : 1$. Find, in terms of $\mathbf{a}$ and $\mathbf{b}$, the position vector of C. [1]

ii. If the angle between $\mathbf{a}$ and $\mathbf{b}$ is $60°$, show that the length of projection of $\overrightarrow{OC}$ on $\overrightarrow{OA}$ is

$$\frac{1}{3}(|\mathbf{a}| + |\mathbf{b}|)$$

[4]