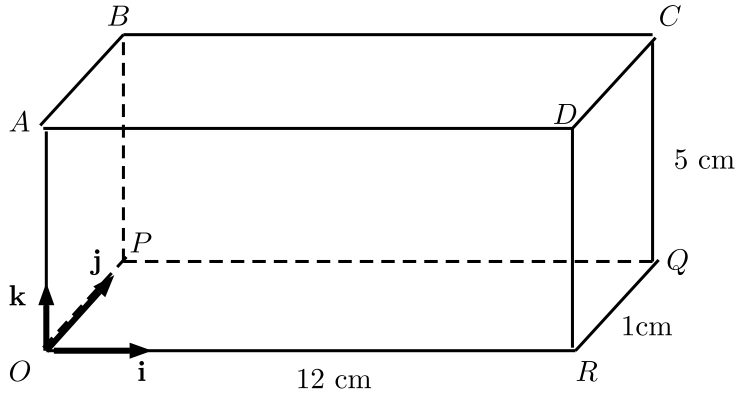
Vectors Revision

1. [2016/SRJC/Prelim/II/2)]



The cuboid above is formed by the eight vertices O, A, B, C, D, P, Q and R. Perpendicular unit vectors $\mathbf{i}, \mathbf{j}, \mathbf{k}$ are parallel to OR, OP and OA respectively.

The length of OR, OP and OA are 12 cm, 1 cm and 5 cm respectively.

i. Find the cartesian equation of line AC. [2]

ii. Find the acute angle between CA and CR. Hence, find the shortest distance from R to AC. [4]

iii. The point T is on AC produced such that $\overrightarrow{AT} = \lambda\overrightarrow{AC}$ and M is the midpoint of OR. The unit vector in the direction of OT is represented by the vector $\overrightarrow{OV}$. By considering the cross product of relevant vectors, find the ratio of the area of triangle OVM to the area of triangle ORT in terms of λ. [3]

[(i) $\frac{x}{12} = y, z = 5$ (ii) 89.1°, 5.10 cm (iii) $1 : 2\sqrt{25 + 145\lambda^2}$]

2. [2015/JJC/Prelim/I/7]

The plane π has equation $13x - 5y - 7z = 0$, and the line l_1 has equation $\frac{x-11}{3} = \frac{y-7}{5} = \frac{z-4}{2}$.

i. Calculate $\begin{pmatrix}3\\5\\2\end{pmatrix} \cdot \begin{pmatrix}13\\-5\\-7\end{pmatrix}$ and state the geometrical relationship between the line l_1 and the plane π. [3]

ii. Given that P is the point (11,7,4), find the acute angle between the line OP and the plane π. [2]

iii. Find the shortest distance from O to l_1. [3]

iv. The line l_2 has equation $\mathbf{r} = \lambda\mathbf{d}$, where $\mathbf{d} \neq 0$. Given that $\begin{pmatrix}3\\5\\2\end{pmatrix} \times \mathbf{d} = \mathbf{0}$, what can be deduced about the lines l_1 and l_2? [1]

[(i) l_1 parallel to π (ii) 22.1° (iii) 5.83 (iv) Parallel]

3. [2015/RI/Prelim/II/1]

Planes P_1, P_2 and P_3 have equations

$$x - 4y - 13z = 3, \qquad -x + \mu y + 5z = -5 \qquad -3x - 4y + z = 1$$

respectively, where μ is a constant. The planes P_1 and P_2 intersect in a line l and are both perpendicular to P_3.

i. Show that $\mu = 2$. [1]

ii. Find a vector equation of l. [2]

The point A with coordinates $(1,-1,0)$ lies on p_3 and the point F is the foot of perpendicular from A to l.

iii. Find the coordinates of F. [1]

iv. Find the exact shortest distance from A to P_1. [2]

[(ii) $\mathbf{r} = \begin{pmatrix} 7 \\ 1 \\ 0 \end{pmatrix} + \alpha \begin{pmatrix} -3 \\ -4 \\ 1 \end{pmatrix}$ (iii) (4,-3,1) (iv) $\frac{2}{\sqrt{186}}$]

[$-y + z = 0$]

4. [2015/IJC/Prelim/I/7]

A line l passes through the points A and B with coordinate (0,-2,2) and (1,0,1) respectively.

i. Find the acute angle between $\overrightarrow{OA}$ and the line l, where O is the origin. [2]

ii. Hence, find the shortest distance from O to the line l, leaving your answer in exact form. [1]

A plane π_1 has equation $\mathbf{r} \cdot (2\mathbf{i} - \mathbf{j}) = 2$.

iii. Show that the line l lies in the plane π_1. [2]

A second plane π_2 contains the line l and is perpendicular to the plane π_1.

iv. Find the cartesian equation of the plane π_2. [2]

v. A third plane π_3 is perpendicular to both π_1 and π_2, and is at a perpendicular distance of $\sqrt{6}$ units from O. Find the possible vector equations of π_3, expressing your answer in the form $\mathbf{r} \cdot \mathbf{n} = d$. [3]

[(i) $30°$ (ii) $\sqrt{2}$ (iv) $x + 2y + 5z = 6$ (v) $\mathbf{r} \cdot \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix} = \pm 6$]

5. [2015/NYJC/Prelim/I/12]

(a) The fixed point A has position vector $\mathbf{a}$ relative to a fixed point O. A variable point P has position vector $\mathbf{r}$ relative to O. Find the locus of P if $\mathbf{r} \cdot (\mathbf{r} - \mathbf{a}) = 0$. [2]

(b) The lines l_1 and l_2 have equations $\mathbf{r} = \begin{pmatrix} 0 \\ 7 \\ 6 \end{pmatrix} + \lambda \begin{pmatrix} -3 \\ 2 \\ 2 \end{pmatrix}$ and $\mathbf{r} = \begin{pmatrix} -3 \\ 6 \\ -4 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ -5 \\ 6 \end{pmatrix}$ respectively, where λ and μ are real parameters.

i. Show that l_1 and l_2 are skew. [2]

ii. The point P_1 on line l_1 and the point P_2 on line l_2 are such that $\overrightarrow{P_1P_2}$ is perpendicular to both l_1 and l_2. By finding a vector normal to l_1 and l_2, or otherwise, find the position vevtors of P_1 and P_2. Show that the length of P_1P_2 is 6 units. [6]

[Locus of P is a sphere with OA as diameter]

6. [2016/PJC/Prelim/I/10)]

The point A has coordinates $(18, 2, 0)$. The plane p_1 has the equation $x + 3y + z = a$, where a is a constant. It is given that p_1 contains the line l_1 with equation $\frac{x-1}{2} = y = \frac{z-1}{-5}$.

i. Show that $a = 2$. [2]

ii. Find the coordinates of the foot of the perpendicular from the point A to p_1. [3]

iii. B is given to be a general point on l_1. Find an expression for the distance between the point A and B. Hence find the position vector of B that is nearest to A. [4]

iv. The planes p_2 and p_3 have the equations $x+z=1$ and $2x+by+z=4$ respectively, where b is a constant.

Given that p_2 and p_3 intersect at l_2, show that l_2 is parallel to the vector $\begin{pmatrix} -b \\ 1 \\ b \end{pmatrix}$. By finding a point that lies on both planes, find a vector equation of l_2. [3]

[(ii) $N(16,-4,-2)$ (iii) $|\overrightarrow{AB}| = \sqrt{294-72\lambda+30\lambda^2}$, $\overrightarrow{OB} = \begin{pmatrix} \frac{56}{15} \\ \frac{41}{30} \\ \frac{-35}{6} \end{pmatrix}$ (iv) $l_2 : r = \begin{pmatrix} 3 \\ 0 \\ -2 \end{pmatrix} + \mu \begin{pmatrix} -b \\ 1 \\ b \end{pmatrix}, \mu \in \mathbb{R}$ (The position vector you choose can be different)]