Complex Numbers

Revision Problem Set

Sourced from all major JC Preliminary Examinations 2025

Q	School	Topic	Marks	Difficulty
Section A	**Solving equations with z^*, $\lvert z\rvert$ or simultaneous**			
1	RVHS/P1	Quadratic with complex coefficients; find $\sqrt{-7-24\mathrm{i}}$	6	●○○
2	VJC/P1	Equation with $\lvert z\rvert$ and z^*; cube roots; equidistant circle	9	●●○
3	TJC/P1	Simultaneous with $\lvert w\rvert$; rotation to build a square on Argand	10	●●○
4	SAJC/P1	Simultaneous with $\lvert z\rvert$; conjugate pair geometry; deduce $\mathrm{Im}(v)=0$	8	●●○
5	MI/P2	Simultaneous with z^*; cubic with real coefficients; root $2+\mathrm{i}$	8	●●○
Section B	**Polynomial roots (real & complex coefficients)**			
6	EJC/P1	Find $\sqrt{2\mathrm{i}}$; polynomial roots; substitution $z \to \mathrm{i}v$	10	●●○
7	TMJC/P1	Real-coefficient cubic; Argand; $\tan\frac{3\pi}{8}$ proof	11	●●●
8	ACJC/P1	Quartic; modulus and argument; parallelogram; cosine rule	13	●●●
9	HCI/P1	$f(z)=z^4-6z^2+k$; symmetry; all roots; reciprocal substitution	12	●●●
10	JPJC/P2	Cubic; $z_3=z_1+z_2$; prove $z_1/z_2=\mathrm{i}$; prove square	11	●●○
11	CJC/P2	Quartic root $3+\mathrm{i}$; student claim; all roots; area	12	●●○
12	SAJC/P2	Polynomial px^6+qx^4+r; cubic with complex coefficient a	6	●●○
13	NYJC/P2	Purely imaginary coefficients; conjugate pair explanation; substitution	6	●○○
Section C	**Argand diagram geometry**			
14	RI/P1	Prove $\arg(kw)=\arg(w)$; plot kw, $\mathrm{i}kw$; iterated $f(z)=z^2-2z$	12	●●●
15	RVHS/P2	Isosceles right-angled triangle by rotation; parallelogram $ABDC$	5	●○○
Section D	**Modulus-argument, loci, geometric properties**			
16	DHS/P1	Ellipse locus; $\arg(z_1+z_2)$; quadratic with complex coefficients	11	●●●
17	NYJC/P1	Rhombus on Argand; reflection of Q in ray OP; express q'	8	●●○
18	ASRJC/P2	Modulus-argument form; rhombus; $\tan\frac{11\pi}{24}$ proof	10	●●○
19	VJC/P2	Unit circle z; $w=\mathrm{i}\sqrt{3}\,z$; geometric; $\tan\frac{5\pi}{12}$ proof	9	●●○
20	NJC/P2	$w=-\mathrm{i}\sqrt{3}$; Argand with w, $-w$, $2-w$; find $\arg(2-w)$	7	●○○

Difficulty: ●○○ Foundational ●●○ Standard ●●● Challenging

Section A · Solving equations with z^*, $|z|$ or simultaneous

Q1 [2025/RVHS/P1/Q3] **[6 marks]**

Do not use a calculator in answering this question.

Find the roots of the equation $z^2 - (1 + 2\text{i})z + 1 + 7\text{i} = 0$, giving your answers in the cartesian form $a + b\text{i}$.

$z = 2 - \text{i} \quad \text{or} \quad z = -1 + 3\text{i}$

Q2 [2025/VJC/P1/Q7] **[9 marks]**

Do not use a calculator in answering this question.

(a) Find the complex number z which satisfies the equation $\dfrac{4|z|}{15 - z^*} = 5\text{i}$. [3]

(b) The complex number w is such that $(w - \text{i})^3 = -\text{i}$.

(i) Given that one possible value of w is 2i, find the two other possible values of w. Give your answers in cartesian form $a + b\text{i}$. [4]

(ii) The points W_1, W_2 and W_3 on the Argand diagram represent the three roots of the equation $(w - \text{i})^3 = -\text{i}$, and the point A represents the complex number $k\text{i}$, where k is a positive real number. Show that the points W_1, W_2 and W_3 lie on a circle with centre A on the Argand diagram for some value of k, stating the value of k. [2]

$z = 15 - 20\text{i}$; $w = \pm\frac{\sqrt{3}}{2} + \frac{1}{2}\text{i}$; $k = 1$

Q3 [2025/TJC/P1/Q8] **[10 marks]**

Do not use a calculator in answering this question.

(a) The complex numbers z and w satisfy the following equations.

$$2z + |w| = 2 - 4\text{i} \qquad \text{i}z - w = 2\text{i}$$

Find z and w, giving your answers in the form $a + b\text{i}$. [5]

(b) It is given that $z_1 = 2 - \text{i}$ and $z_3 = -3 + 2\text{i}$.

(i) On an Argand diagram, mark the points A and C representing z_1 and z_3 respectively. [1]

(ii) The points B and D represent z_2 and z_4 respectively. Given that $ABCD$ is a square labelled in an anti-clockwise direction, find z_2 and z_4. [4]

$z = -\frac{2}{3} - 2\text{i}$, $w = 2 - \frac{8}{3}\text{i}$; $z_2 = 1 + 3\text{i}$, $z_4 = -2 - 2\text{i}$

Q4 [2025/SAJC/P1/Q10] **[8 marks]**

Do not use a calculator in answering this question.

(a) Find the complex numbers z and w which satisfy the following simultaneous equations.

$$|z| + 5w = 0 \qquad \text{i}z - 4w = -4 + 7\text{i}$$

Give your answers in the form $a + b\text{i}$. [5]

(b) The point A on the Argand diagram represents the complex number u.

(i) On the Argand diagram, plot the point B to represent $-u$. [1]

(ii) The points C and D represent $v-u$ and $(v-u)^*$ respectively, where v is an unknown complex number. It is given that $\angle CDA = 90°$. State the value of $\text{Im}(v)$ and justify your answer. [2]

$z = 7 + 24\text{i},\ w = -5;\ \text{Im}(v) = 0$

Q5 [2025/MI/P2/Q3] **[8 marks]**

Do not use a calculator in answering this question.

(a) The complex numbers z and w satisfy the simultaneous equations

$$z + 2z^* + w = 6 + \text{i} \qquad \text{i}z - w = 3.$$

Find z and w, leaving your answers in the form $a + b\text{i}$. [4]

(b) The diagram shows the curve with equation $y = \text{f}(x)$, where $\text{f}(x) = x^3 + mx^2 + nx + 5$, and m and n are real constants. The curve crosses the x-axis at the point P. One of the roots of the equation $\text{f}(x) = 0$ is $2 + \text{i}$. Determine the coordinates of P. [4]

$z = 4 + 3\text{i},\ w = -6 + 4\text{i};\ P = (-1, 0)$

Section B · Polynomial roots (real & complex coefficients)

Q6 [2025/EJC/P1/Q8] **[10 marks]**

Do not use a calculator in answering this question.

The complex number w is such that $w^2 = 2\text{i}$ and $0 \le \arg(w) \le \dfrac{\pi}{2}$.

(a) Find w. [3]

(b) Given that one of the roots of $z^3 - z^2 + (1-\text{i})z + s = 0$ is w, find the other roots and the value of s. [5]

(c) Hence, find the roots of $-\text{i}v^3 + v^2 + (1+\text{i})v + s = 0$. [2]

$w = 1 + \text{i};$ other roots: $0, -\text{i};\ s = 0$

Q7 [2025/TMJC/P1/Q9] **[11 marks]**

Do not use a calculator in answering this question.

It is given that $-2 + 2\text{i}$ is a root of $z^3 + az^2 + bz - 16\sqrt{2} = 0$, where a and b are real.

(a) Find the values of a and b and the other two roots. [5]

(b) In an Argand diagram with origin O, the three roots are represented by points A, B and C where A represents $-2 + 2\text{i}$ and C represents the real root. Label these points on an Argand diagram, indicating clearly the modulus and argument of each root. State also a geometrical relationship between A and B. [3]

(c) Hence, prove that $\tan\dfrac{3\pi}{8} = 1 + \sqrt{2}$. [3]

$a = 4 - 2\sqrt{2},\ b = 8 - 8\sqrt{2};$ roots:
$-2 \pm 2\text{i}, 2\sqrt{2}$

Q8 [2025/ACJC/P1/Q10] **[13 marks]**

Do not use a calculator in answering this question.

(a) One of the roots of $\omega^4 - 2\omega^3 + 10\omega^2 + p\omega + q = 0$, where p and q are real, is $2 + 3\text{i}$. Find the values of p and q and the other roots. [6]

(b) The complex numbers u and v are such that $u = -\sqrt{2} + \mathrm{i}\sqrt{2}$, and $|v| = 3$ and $\arg v = \theta$, where $0 < \theta < \dfrac{\pi}{4}$. Points A, B and C represent u, v and $u+v$ respectively.

(i) Find the modulus and argument of u. [2]

(ii) Sketch the points A, B and C on an Argand diagram. [2]

(iii) By finding $\angle OAC$ or otherwise, show that $|u+v|^2 = a + b\cos(\theta + K)$, where a, b and K are constants to be determined. [3]

$p = 6,\ q = 65;$ other roots: $2 - 3\mathrm{i}, -1 \pm 2\mathrm{i};\ a = 13, b = -12, K = \frac{\pi}{4}$

Q9 [2025/HCI/P1/Q12] **[12 marks]**

Do not use a graphing calculator for this question.

It is given that $\mathrm{f}(z) = z^4 - 6z^2 + k$, where k is a non-zero constant.

(a) If k is purely imaginary, determine with justification whether $\mathrm{f}(z) = 0$ can have real roots. [1]

(b) Show that $\mathrm{f}(-z) = \mathrm{f}(z)$. [1]

(c) Given that $2 + \mathrm{i}$ is a root of $\mathrm{f}(z) = 0$, determine k. Hence find the remaining roots. [6]

(d) Given the product of all roots of $\mathrm{f}(z) = 0$ is D, find D. [2]

(e) A complex number w_1 satisfies $kw^4 - 6w^2 + 1 = 0$. Given that w_1 can be obtained from $2 + \mathrm{i}$, find w_1. [2]

$k = 25;$ roots: $\pm 2 \pm \mathrm{i};\ D = 25;\ w_1 = \frac{2}{5} - \frac{1}{5}\mathrm{i}$

Q10 [2025/JPJC/P2/Q4] **[11 marks]**

Do not use a calculator in answering this question.

(a) One of the roots of $5w^3 + pw^2 + 68w + q = 0$, where p and q are real, is $3 - \mathrm{i}$. Find the other roots and the values of p and q. [5]

(b) Two complex numbers are given by $z_1 = -1 + 2\mathrm{i}$ and $z_2 = 2 + \mathrm{i}$. Draw an Argand diagram showing z_1 and z_2 with $z_3 = z_1 + z_2$ marked, showing the geometrical relationship. [2]

(i) Find $\dfrac{z_1}{z_2}$ in the form $k\mathrm{i}$. [2]

(ii) Hence show that $OZ_2Z_3Z_1$ is a square. [2]

$p = -33,\ q = -30;$ other roots: $3 + \mathrm{i}, \frac{3}{5};\ \frac{z_1}{z_2} = \mathrm{i}$

Q11 [2025/CJC/P2/Q4] **[12 marks]**

One of the roots of $2z^4 - 14z^3 + 33z^2 - 26z + p = 0$, where p is a constant, is $3 + \mathrm{i}$.

(a) Based on the above information only, a student claims the equation has root $3 - \mathrm{i}$. State with a reason why this may not be true. [1]

(b) Show that $p = 10$. [2]

(c) (Do not use a calculator.) Find all roots of $2z^4 - 14z^3 + 33z^2 - 26z + 10 = 0$ and mark them on a labelled Argand diagram. [7]

(d) Identify the type of quadrilateral formed and determine its area. [2]

Roots: $3 \pm \mathrm{i}, \frac{1}{2} \pm \frac{1}{2}\mathrm{i};$ Trapezium, area $= \frac{15}{4}$

Q12 [2025/SAJC/P2/Q2] **[6 marks]**

(a) It is given that $f(x) = px^6 + qx^4 + r$, where p, q, r are real constants.

(i) Show that if $x = \alpha$ is a root of $f(x) = 0$, then $x = -\alpha$ is also a root. [1]

(ii) Given that $x = 5$ and $x = \beta$ are roots where $\mathrm{Re}(\beta) > 0$ and $\mathrm{Im}(\beta) > 0$, write down all remaining roots. [3]

(b) The complex number z satisfies $z^3 + (3 - a)z^2 - (2 + 6\mathrm{i})z - 6 = 0$, where a is complex and one root is $1 + \mathrm{i}$. Find a and the other roots. [5]

Remaining: $-5, -\beta, \beta^*, -\beta^*$; $a = 2\mathrm{i}$; other roots: $-3, -1 + \mathrm{i}$

Q13 [2025/NYJC/P2/Q2] **[6 marks]**

(a) It is given that the roots of $-\mathrm{i}x^3 + 5\mathrm{i}x^2 + ax + b = 0$, where a and b are purely imaginary, are $2 + \mathrm{i}$, $2 - \mathrm{i}$ and 1. Explain why the complex roots occur in conjugate pairs. [2]

(b) By using (a) and an appropriate substitution, find the roots of $\mathrm{i}x^3 + 5\mathrm{i}x^2 + a^*x + b = 0$, where a^* denotes the complex conjugate of a. [4]

Roots of new eqn: $-2 - \mathrm{i}$, $-2 + \mathrm{i}$, -1

Section C · Argand diagram geometry

Q14 [2025/RI/P1/Q8] **[12 marks]**

Do not use a calculator in answering this question.

(a) The point P on the Argand diagram represents $w = u + \mathrm{i}v$, where $u, v > 0$.

(i) Explain algebraically why $\arg(kw) = \arg(w)$ for any real constant $k > 1$. [1]

(ii) Points Q and R represent kw and $\mathrm{i}kw$, where $k > 1$ is real. On an Argand diagram, plot P, Q and R, showing clearly the geometrical relationship between them. [2]

(b) In another Argand diagram, points A, B and C represent z, $f(z)$ and $f(f(z))$ where $f(z) = z^2 - 2z$.

(i) Show that $f(f(z)) - f(z) = z(z - 2)(z - 3)(z + 1)$. [2]

(ii) ABC is a right-angled triangle (anticlockwise, right angle at B) with $BC = m\,BA$, $m > 0$ real. By considering $\dfrac{f(f(z)) - f(z)}{f(z) - z}$ or otherwise, show that $(z - 2)(z + 1) = m\mathrm{i}$. [2]

(iii) In the case where $z = x + 2\mathrm{i}$ with $x > 0$, find x and m. Hence obtain the complex number represented by B. [5]

$x = 3$, $m = 10$; $B = -1 + 8\mathrm{i}$

Q15 [2025/RVHS/P2/Q2] **[5 marks]**

Points A, B and C represent $z_A = 5 + 6\mathrm{i}$, $z_B = 9 + 3\mathrm{i}$ and z_C respectively. ABC is an isosceles triangle labelled clockwise with $\angle CAB = 90°$.

(a) Find z_C. [3]

(b) The point D representing z_D is such that $ABDC$ is a parallelogram. Find z_D. [2]

$z_C = 2 + 2\mathrm{i}$; $z_D = 6 - \mathrm{i}$

Section D · Modulus-argument, loci, geometric properties

Q16 [2025/DHS/P1/Q8] **[11 marks]**

Do not use a calculator in answering this question.

A complex number z varies with t such that $z = 2\cos t + \mathrm{i}(3\sin t)$, where $0 \le t < 2\pi$.

(a) By taking $x = \mathrm{Re}(z)$ and $y = \mathrm{Im}(z)$, sketch on an Argand diagram the curve showing the positions of z. Find the cartesian equation of this curve. [3]

(b) Two complex numbers z_1 and z_2 for two distinct values of t satisfy $|z_1| = |z_2|$ and $0 < \arg(z_1) < \dfrac{\pi}{2}$. By referring to the Argand diagram in (a), find the possible values of $\arg(z_1 + z_2)$. [2]

(c) It is given that z_1 and z_2 are roots of $z^2 + \alpha z + \beta = 0$. Explain whether α must be real. [2]

(d) Given that α is not real and $|z_1| = |z_2| = \dfrac{\sqrt{26}}{2}$, find α and β. [4]

$\alpha = -3\sqrt{2}\,\mathrm{i};\ \beta = -\frac{13}{2}$

Q17 [2025/NYJC/P1/Q6] **[8 marks]**

Points P, Q and R represent complex numbers p, q and r with $\arg(p) = \alpha$, $\arg(q) = \beta$, where $0 < \alpha < \beta/2$, $\beta > 2\alpha$, and $r = p + q$.

(a) If $|p| = |q|$, describe the shape of quadrilateral $OPRQ$. Hence find $\arg(r)$ in terms of α and β.[3]

(b) The point Q' representing q' is the reflection of Q in OP. State the angle POQ'. [1]

By leaving answers in terms of α, β and $|q|$ where applicable:

(i) Find $\arg(q')$. [1]

(ii) Find the real and imaginary parts of q' and write q' in $a + b\mathrm{i}$ form. [3]

Rhombus; $\arg(r) = \frac{1}{2}(\alpha + \beta)$; $\arg(q') = 2\alpha - \beta$

Q18 [2025/ASRJC/P2/Q6] **[10 marks]**

The points P and Q are represented by $p = \sqrt{2} + \sqrt{2}\,\mathrm{i}$ and q where $\arg(q) = \dfrac{2\pi}{3}$, $|q| = 2$.

(a) Find $|p|$ and $\arg(p)$ in exact form. [2]

(b) Sketch P and Q on an Argand diagram. [2]

(c) Use the Argand diagram to deduce $\mathrm{Re}(q)$ and $\mathrm{Im}(q)$ in exact form. [2]

(d) The point R represents $p + q$. What is the shape of quadrilateral $OPRQ$? [1]

(e) By considering $\arg(p + q)$ or otherwise, show that $\tan \dfrac{11\pi}{24} = \sqrt{6} + \sqrt{3} + \sqrt{2} + 2$. [3]

$|p| = 2$; $\arg(p) = \frac{\pi}{4}$; Rhombus

Q19 [2025/VJC/P2/Q4] **[9 marks]**

Do not use a calculator in answering this question.

The complex number z has modulus 1 and argument θ, where $\dfrac{\pi}{2} < \theta < \pi$, and $w = \mathrm{i}\sqrt{3}\,z$. Point P represents z.

(a) On an Argand diagram, plot Q and R representing w and $z - w$ respectively. Show the geometrical relationship between P, Q and R. [3]

(b) Find the area of quadrilateral $ORPQ$. [1]

Given $\theta = \dfrac{3\pi}{4}$:

(c) Find z in the form $x + y\mathrm{i}$. [2]

(d) Show that $z - w = k[(\sqrt{3} - 1) + (\sqrt{3} + 1)\mathrm{i}]$ for a constant k to be determined. [2]

(e) Hence show that $\tan\dfrac{5\pi}{12} = \dfrac{\sqrt{3}+1}{\sqrt{3}-1}$. [1]

Area $= \sqrt{3};\ k = \frac{1}{\sqrt{2}};\ z = -\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\mathrm{i}$

Q20 [2025/NJC/P2/Q1] **[7 marks]**

Do not use a calculator in answering this question.

The complex number w is such that $w = -\mathrm{i}\sqrt{3}$.

(a) Find $|w|$ and $\arg(w)$ in exact form. [2]

(b) Represent w, $-w$ and $2 - w$ on the same Argand diagram, illustrating clearly the geometrical relationship between them. [3]

(c) Hence, find $\arg(2 - w)$ exactly. [2]

$|w| = 2;\ \arg(w) = \frac{5\pi}{6};\ \arg(2-w) = -\frac{\pi}{12}$