

Complex Numbers

Worked Solutions

2025 JC Preliminary Examinations

20 questions · Sourced from all major JC Preliminary Examinations 2025

- Section A: Equations with z^* , $|z|$, simultaneous
 ■ Section B: Polynomial roots
■ Section C: Argand diagram geometry
 ■ Section D: Modulus-argument, loci

How to use these solutions

These worked solutions are intended for *review after attempting the questions independently*. Each solution follows the official marking scheme logic verbatim. Examiner notes highlight the most common errors seen in marking.

••• Foundational •• Standard ••• Challenging

Q	Source	Topic / Subtopic	Diff.
■ Section A Solving equations with z^* , $ z $ or simultaneous			
1	RVHS 2025 P1	Quadratic with complex coefficients; find $\sqrt{-7-24i}$	•••
2	VJC 2025 P1	Equation with $ z $ and z^* ; cube roots; equidistant circle	•••
3	TJC 2025 P1	Simultaneous with $ w $; rotation to build a square on Argand	•••
4	SAJC 2025 P1	Simultaneous with $ z $; conjugate pair geometry; $\text{Im}(v) = 0$	•••
5	MI 2025 P2	Simultaneous with z^* ; cubic; real coefficients; root $2+i$	•••
■ Section B Polynomial roots (real & complex coefficients)			
6	EJC 2025 P1	Find $\sqrt{2i}$; polynomial roots; substitution $z \rightarrow iv$	•••
7	TMJC 2025 P1	Real-coefficient cubic; Argand; $\tan \frac{3\pi}{8}$ proof	•••
8	ACJC 2025 P1	Quartic; modulus and argument; parallelogram; cosine rule	•••
9	HCI 2025 P1	$f(z) = z^4 - 6z^2 + k$; symmetry; all roots; reciprocal substitution	•••
10	JPJC 2025 P2	Cubic; $z_3 = z_1 + z_2$; prove $z_1/z_2 = i$; prove square	•••
11	CJC 2025 P2	Quartic root $3+i$; student claim; all roots; quadrilateral area	•••
12	SAJC 2025 P2	Polynomial $px^6 + qx^4 + r$; cubic with complex coefficient a	•••
13	NYJC 2025 P2	Purely imaginary coefficients; conjugate pair explanation; substitution	•••
■ Section C Argand diagram geometry			
14	RI 2025 P1	Prove $\arg(kw) = \arg(w)$; iterated $f(z) = z^2 - 2z$; right-angle triangle	•••
15	RVHS 2025 P2	Isosceles right-angled triangle by rotation; parallelogram $ABDC$	•••
■ Section D Modulus-argument, loci, geometric properties			
16	DHS 2025 P1	Ellipse locus; $\arg(z_1 + z_2)$; quadratic with complex coefficients	•••
17	NYJC 2025 P1	Rhombus on Argand; reflection of Q in ray OP ; express q'	•••
18	ASRJC 2025 P2	Modulus-argument form; rhombus; $\tan \frac{11\pi}{24}$ proof	•••
19	VJC 2025 P2	Unit circle z ; $w = i\sqrt{3}z$; geometric; $\tan \frac{5\pi}{12}$ proof	•••
20	NJC 2025 P2	$w = -i\sqrt{3}$; Argand with w , $-w$, $2-w$; find $\arg(2-w)$	•••

Section A Equations with z^* , $|z|$ or Simultaneous

Q1 – Q5

Q1 RVHS 2025 Paper 1 Q3

••• [6 marks]

In the above equation, $a = 1$, $b = -1 - 2i$, $c = 1 + 7i$

$$z = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{1 + 2i \pm \sqrt{(-1 - 2i)^2 - 4(1)(1 + 7i)}}{2} = \frac{1 + 2i \pm \sqrt{-7 - 24i}}{2}$$

We find $\sqrt{-7 - 24i}$. Let $\sqrt{-7 - 24i} = x + yi$, $x, y \in \mathbb{R}$.

Comparing real and imaginary parts of $(x + yi)^2 = -7 - 24i$:

$$x^2 - y^2 = -7 \quad (1) \quad 2xy = -24 \implies x = \frac{-12}{y} \quad (2)$$

Substitute (2) into (1):

$$\begin{aligned} \left(\frac{-12}{y}\right)^2 - y^2 &= -7 \\ y^4 - 7y^2 - 144 &= 0 \\ (y^2 - 16)(y^2 + 9) &= 0 \end{aligned}$$

$$y^2 = 16 \implies y = \pm 4 \quad (y^2 = -9 \text{ rejected, } y \in \mathbb{R})$$

When $y = 4$, $x = -3$; when $y = -4$, $x = 3$. So $\sqrt{-7 - 24i} = \pm(3 - 4i)$.

$$z = \frac{1 + 2i + (3 - 4i)}{2} = 2 - i \quad \text{or} \quad z = \frac{1 + 2i - (3 - 4i)}{2} = -1 + 3i$$

Q2 VJC 2025 Paper 1 Q7

••• [9 marks]

(a) Let $z = a + bi$, $a, b \in \mathbb{R}$.

$$4\sqrt{a^2 + b^2} = 5i[15 - (a - bi)] = -5b + (75 - 5a)i$$

Comparing imaginary parts: $a = 15$. Comparing real parts: $4\sqrt{225 + b^2} = -5b$.

$$16(225 + b^2) = 25b^2 \implies 9b^2 = 3600 \implies b^2 = 400$$

$$b = -20 \quad (\because b < 0 \text{ by equation (1)})$$

$$z = 15 - 20i$$

(b)(i) Expanding $(w - i)^3 = -i$:

$$w^3 - 3iw^2 - 3w + 2i = 0 = (w - 2i)(w^2 - iw - 1)$$

Solving $w^2 - iw - 1 = 0$:

$$w = \frac{i \pm \sqrt{3}}{2}$$

$$w = \frac{\sqrt{3}}{2} + \frac{1}{2}i \quad \text{or} \quad w = -\frac{\sqrt{3}}{2} + \frac{1}{2}i$$

(b)(ii) Let A represent ki . Setting $|W_1A| = |W_2A|$:

$$\left(\frac{\sqrt{3}}{2}\right)^2 + \left(\frac{1}{2} - k\right)^2 = (2 - k)^2 \implies k = 1$$

Check: $|W_1A| = |2i - i| = 1$, $|W_2A| = \left|\frac{\sqrt{3}+i}{2} - i\right| = 1$, $|W_3A| = \left|\frac{-\sqrt{3}-i}{2}\right| = 1$. All equal 1.
 W_1, W_2, W_3 lie on a circle of radius 1 centred at i , with $k = 1$.

Q3 TJC 2025 Paper 1 Q8

••• [10 marks]

(a) From $iz - w = 2i$: $w = iz - 2i$. Let $w = a + bi$. After substitution and comparing real/imaginary parts, $a = 2$ and solving gives:

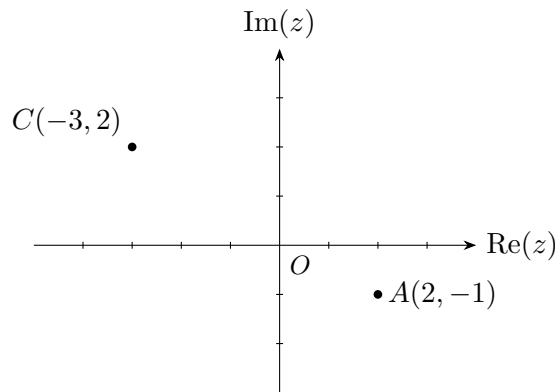
$$9b^2 + 200b - (9 \cdot \frac{64}{9} + 200 \cdot (-\frac{8}{3})) = 0$$

$$b = -\frac{8}{3} \quad (\text{rejecting } b = 0 \text{ since } \sqrt{4 + b^2} \neq -2)$$

$$|w| = \sqrt{4 + \frac{64}{9}} = \frac{10}{3} \implies z = \frac{2 - 4i - \frac{10}{3}}{2} = -\frac{2}{3} - 2i$$

$$z = -\frac{2}{3} - 2i, \quad w = 2 - \frac{8}{3}i$$

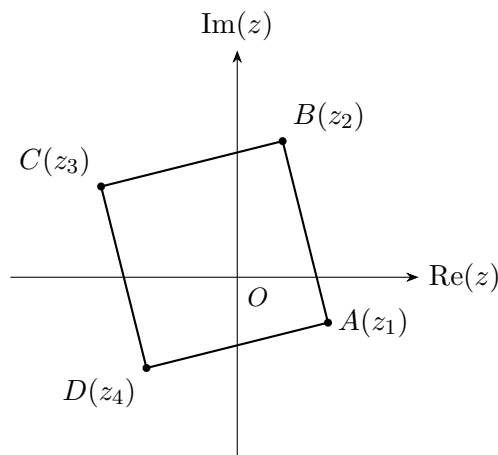
(b)(i)



(b)(ii) Since $ABCD$ is anticlockwise with $\angle ABC = \frac{\pi}{2}$, A is C rotated 90° anticlockwise about B :

$$i(z_3 - z_2) = z_1 - z_2 \implies (1 - i)z_2 = 4 + 2i \implies z_2 = \frac{(4 + 2i)(1 + i)}{2} = 1 + 3i$$

$$z_4 = z_3 + (z_2 - z_1) \cdot (\text{via } \overrightarrow{AB} = \overrightarrow{DC}) = -3 + 2i + (1 + 3i) - (2 - i) = -2 - 2i$$



$$z_2 = 1 + 3i, \quad z_4 = -2 - 2i$$

Q4 SAJC 2025 Paper 1 Q10

••• [8 marks]

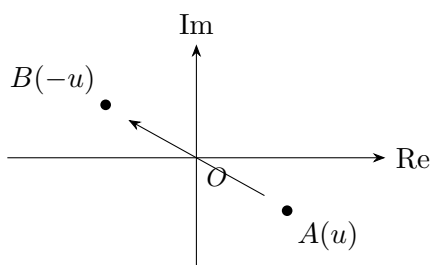
(a) From $|z| + 5w = 0$: $w = -\frac{1}{5}|z|$. Substituting into $iz - 4w = -4 + 7i$, let $z = x + yi$:

$$\left(-y + \frac{4}{5}\sqrt{x^2 + y^2}\right) + xi = -4 + 7i$$

Comparing: $x = 7$. Then $-y + \frac{4}{5}\sqrt{49 + y^2} = -4$, giving $9y^2 - 200y - 384 = 0$, so $(y - 24)(9y + 16) = 0$.
 $y = -\frac{16}{9}$ rejected (fails sign check), hence $y = 24$.

$$z = 7 + 24i, \quad w = -\frac{1}{5}\sqrt{625} = -5$$

(b)(i) B is the reflection of A through the origin (represents $-u$).



(b)(ii) C and D are a conjugate pair so CD is vertical. With $\angle CDA = 90^\circ$ and CD vertical, DA must be horizontal. Since A and B are symmetric about the real axis, BC is also horizontal. Since C represents $v - u$, vector \overrightarrow{BC} represents v . As BC is horizontal, v is real.

$$\text{Im}(v) = 0$$

Q5 MI 2025 Paper 2 Q3

••• [8 marks]

(a) From $iz - w = 3$: $w = iz - 3$. Substituting into $z + 2z^* + w = 6 + i$:

$$(1 + i)z + 2z^* = 9 + i$$

Let $z = a + bi$: $(3a - b) + (a - b)i = 9 + i$, giving $a = 4$, $b = 3$.

$$z = 4 + 3i, \quad w = i(4 + 3i) - 3 = -6 + 4i$$

(b) All coefficients of $f(x)$ are real and $2 + i$ is a root, so $2 - i$ is also a root.

$$[x - (2 + i)][x - (2 - i)] = x^2 - 4x + 5$$

$$f(x) = (x + c)(x^2 - 4x + 5) \implies 5c = 5 \implies c = 1$$

$$x = -1 \text{ or } x = 2 \pm i, \quad \text{so } P = (-1, 0).$$

Section B Polynomial Roots

Q6-Q13

Q6 EJC 2025 Paper 1 Q8

••• [10 marks]

(a) Let $w = a + bi$. Then $a^2 - b^2 = 0$ and $2ab = 2$, giving $a = 1/b$ and $1 - b^4 = 0$, so $b^2 = 1$.
 $b = -1$, $a = -1$ rejected since $0 \leq \arg(w) \leq \pi/2$.

$$w = 1 + i$$

(b) Write $z^3 - z^2 + (1 - i)z + s = [z - (1 + i)](z^2 + Bz + C)$.

Comparing coefficients:

$$z^2: \quad -1 = -1 - i + B \implies B = i$$

$$z: \quad 1 - i = -(1 + i)i + C = 1 - i + C \implies C = 0$$

$$\text{constant: } s = -(1 + i)(0) = 0$$

$$z^2 + iz = z(z + i) = 0$$

$$\text{Other roots: } z = 0 \text{ and } z = -i; \quad s = 0$$

(c) Replace z with iv : $-iv^3 + v^2 + (1 + i)v + s = 0$. So $v = -iz$:

$$v = -i(1 + i) = 1 - i, \quad v = 0, \quad v = -i(-i) = -1$$

Q7 TMJC 2025 Paper 1 Q9

●●● [11 marks]

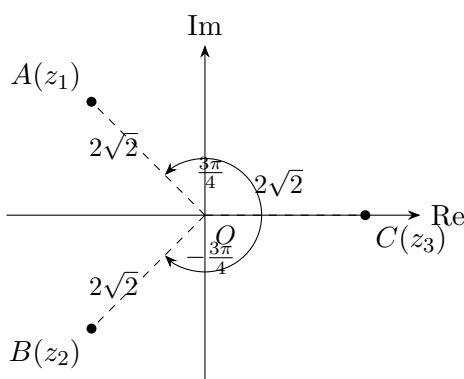
(a) Since coefficients are real and $-2 + 2i$ is a root, $-2 - 2i$ is also a root.

$$\begin{aligned} [z - (-2 + 2i)][z - (-2 - 2i)] &= z^2 + 4z + 8 \\ z^3 + az^2 + bz - 16\sqrt{2} &= (z^2 + 4z + 8)(z - 2\sqrt{2}) \end{aligned}$$

Comparing: $a = 4 - 2\sqrt{2}$, $b = 8 - 8\sqrt{2}$.

$$a = 4 - 2\sqrt{2}, \quad b = 8 - 8\sqrt{2}; \quad \text{roots: } -2 \pm 2i, 2\sqrt{2}$$

(b) $|z_1| = |z_2| = 2\sqrt{2}$, $\arg(z_1) = \frac{3\pi}{4}$, $\arg(z_2) = -\frac{3\pi}{4}$, $|z_3| = 2\sqrt{2}$, $\arg(z_3) = 0$.



B is the reflection of A about the real axis.

(c) Since $OA = OC = 2\sqrt{2}$, let D be such that $OADC$ is a rhombus. Then OD bisects $\angle AOC$, so $\arg(z_D) = \frac{3\pi}{8}$.

$$\begin{aligned} z_D &= z_1 + z_3 = (2\sqrt{2} - 2) + 2i \\ \tan \frac{3\pi}{8} &= \frac{\text{Im}(z_D)}{\text{Re}(z_D)} = \frac{2}{2\sqrt{2} - 2} = \frac{1}{\sqrt{2} - 1} \cdot \frac{\sqrt{2} + 1}{\sqrt{2} + 1} = 1 + \sqrt{2} \quad \square \end{aligned}$$

Q8 ACJC 2025 Paper 1 Q10

[13 marks]

(a) Since $2 - 3i$ is also a root (real coefficients):

$$(\omega^2 - 4\omega + 13)(\omega^2 + a\omega + b) = \omega^4 - 2\omega^3 + 10\omega^2 + p\omega + q$$

Comparing ω^3 : $a = 2$. Comparing ω^2 : $b = 5$.

$$p = -4(5) + 13(2) = 6 \quad q = 13 \times 5 = 65$$

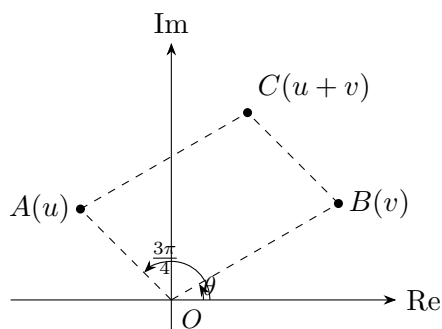
Remaining roots from $\omega^2 + 2\omega + 5 = 0$: $\omega = -1 \pm 2i$.

$$p = 6, q = 65; \quad \text{other roots: } 2 - 3i, -1 + 2i, -1 - 2i$$

(b)(i)

$$|u| = \sqrt{2+2} = 2 \quad \arg(u) = \pi - \frac{\pi}{4} = \frac{3\pi}{4}$$

$$|u| = 2, \quad \arg(u) = \frac{3\pi}{4}$$

(b)(ii) Sketch ($|v| = 3, 0 < \theta < \pi/4$):(b)(iii) $\angle AOB = \frac{3\pi}{4} - \theta$, so $\angle OAC = \pi - (\frac{3\pi}{4} - \theta) = \theta + \frac{\pi}{4}$.By cosine rule on $\triangle OAC$ ($OA = 2, AC = |v| = 3$):

$$|u + v|^2 = 4 + 9 - 12 \cos(\theta + \frac{\pi}{4}) = 13 - 12 \cos(\theta + \frac{\pi}{4})$$

$$a = 13, b = -12, K = \frac{\pi}{4}$$

Q9 HCI 2025 Paper 1 Q12

●●● [12 marks]

(a) If $f(z) = 0$ has real root x , then $x^4 - 6x^2 = -k$ (real = purely imaginary): contradiction.

$f(z) = 0$ **cannot** have real roots.

(b) $f(-z) = (-z)^4 - 6(-z)^2 + k = z^4 - 6z^2 + k = f(z)$. (Shown)

(c)

$$(2 + i)^2 = 3 + 4i$$

$$(2 + i)^4 = (3 + 4i)^2 = -7 + 24i$$

$$-7 + 24i - 6(3 + 4i) + k = 0 \implies k = 25$$

Since coefficients are real, $2 - i$ is also a root. Since $f(-z) = f(z)$, negatives are also roots.

$k = 25$; all roots: $2 + i, 2 - i, -2 + i, -2 - i$

(d)

$$D = (2 + i)(2 - i)(-2 + i)(-2 - i) = 5 \times 5 = 25$$

$D = 25$

(e) $kw^4 - 6w^2 + 1 = 0$ is obtained from $z^4 - 6z^2 + k = 0$ via $w = \frac{1}{z}$, so $w_1 = \frac{1}{2+i}$.

$$w_1 = \frac{2 - i}{(2 + i)(2 - i)} = \frac{2 - i}{5}$$

$w_1 = \frac{2}{5} - \frac{1}{5}i$

Q10 JPJC 2025 Paper 2 Q4

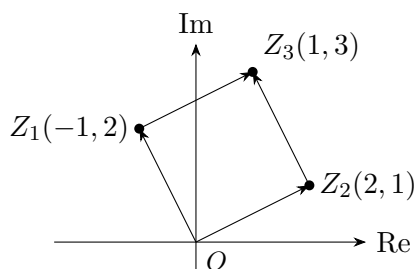
••• [11 marks]

(a) Real coefficients, so $3 + i$ is also a root. $(w^2 - 6w + 10)(5w + a) = 5w^3 + pw^2 + 68w + q$.

Coefficient of w : $-6a + 50 = 68 \implies a = -3$. Then $p = -33$, $q = -30$.

Other roots: $3 + i$, $\frac{3}{5}$; $p = -33$, $q = -30$

(b) Argand diagram with $z_1 = -1 + 2i$, $z_2 = 2 + i$, $z_3 = z_1 + z_2 = 1 + 3i$:



(b)(i)

$$\frac{z_1}{z_2} = \frac{(-1 + 2i)(2 - i)}{5} = \frac{5i}{5} = i$$

$$\frac{z_1}{z_2} = i$$

(b)(ii) $z_1 = iz_2$: rotating Z_2 by $\frac{\pi}{2}$ anticlockwise about O gives Z_1 .

(1) $\angle Z_1 O Z_2 = 90^\circ$. (2) $OZ_1 = OZ_2 = \sqrt{5}$. (3) $\overrightarrow{OZ_3} = \overrightarrow{OZ_1} + \overrightarrow{OZ_2}$, so $OZ_1 Z_3 Z_2$ is a parallelogram.

From (1)+(2)+(3): $OZ_2 Z_3 Z_1$ is a square.

(Shown)

Q11 CJC 2025 Paper 2 Q4

••• [12 marks]

(a) The student's claim may not be true as p may not be real (conjugate root theorem requires real coefficients).

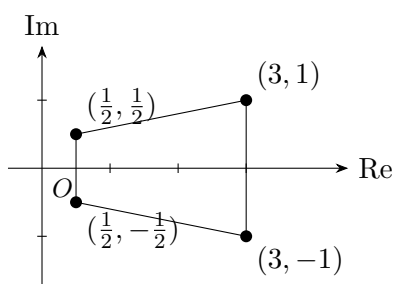
(b) Substituting $z = 3 + i$: $(3 + i)^2 = 8 + 6i$, $(3 + i)^3 = 18 + 26i$, $(3 + i)^4 = 28 + 96i$.

$$2(28 + 96i) - 14(18 + 26i) + 33(8 + 6i) - 26(3 + i) + p = -10 + p = 0 \implies p = 10. \quad \square$$

(c) Coefficients now real, so $3 - i$ is also a root.

$$2z^4 - 14z^3 + 33z^2 - 26z + 10 = (z^2 - 6z + 10)(2z^2 - 2z + 1)$$

$$2z^2 - 2z + 1 = 0 \implies z = \frac{1}{2} \pm \frac{1}{2}i$$



Roots: $3 \pm i$, $\frac{1}{2} \pm \frac{1}{2}i$

(d) The quadrilateral is a **trapezium** (one pair of parallel sides, both vertical at $\text{Re}(z) = \frac{1}{2}$ and $\text{Re}(z) = 3$).

$$\text{Area} = \frac{1}{2}(1 + 2) \times 2.5 = \frac{15}{4} \text{ units}^2$$

Trapezium; Area = $\frac{15}{4}$ units²

Q12 SAJC 2025 Paper 2 Q2

••• [6 marks]

(a)(i) Since $f(\alpha) = p\alpha^6 + q\alpha^4 + r = 0$:

$$f(-\alpha) = p(-\alpha)^6 + q(-\alpha)^4 + r = p\alpha^6 + q\alpha^4 + r = 0 \implies x = -\alpha \text{ is a root. } \square$$

(a)(ii) From (i), $x = -5$ and $x = -\beta$ are also roots. Since f has real coefficients, $x = \beta^*$ and $x = -\beta^*$ are also roots.

Remaining roots: $-5, -\beta, \beta^*, -\beta^*$

(b) Write $z^3 + (3-a)z^2 - (2+6i)z - 6 = (z-1-i)(z^2 + Bz + C)$.

Constant: $-6 = C(-1-i) \implies C = 3 - 3i$.

Coefficient of z : $(1+i)B = 5 + 3i \implies B = \frac{(5+3i)(1-i)}{2} = 4 - i$.

Coefficient of z^2 : $a = 4 - (4-i) + i = 2i$.

$$(z-1-i)(z^2 + (4-i)z + 3-3i) = (z-1-i)(z+3)(z+1-i) = 0$$

$a = 2i$; other roots: $-3, -1+i$

Q13 NYJC 2025 Paper 2 Q2

••• [6 marks]

(a) Let $a = \lambda i$, $b = \mu i$, $\lambda, \mu \in \mathbb{R}$. Dividing through by $-i$:

$$x^3 - 5x^2 - \lambda x - \mu = 0$$

All coefficients are real, so complex roots occur in conjugate pairs.

(Shown)

(b) Substituting $x \rightarrow -x$ into the equation from (a) recovers the form $ix^3 + 5ix^2 + a^*x + b = 0$ (using $a^* = -a$).

So roots are $-x$ for $x \in \{2 + i, 2 - i, 1\}$:

$$x = -2 - i, \quad -2 + i, \quad -1$$

Section C Argand Diagram Geometry

Q14, Q15

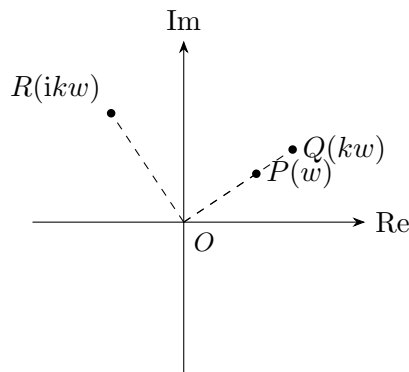
Q14 RI 2025 Paper 1 Q8

••• [12 marks]

(a)(i) Since $w = u + iv$ with $u, v > 0$: $kw = ku + kvi$ with $ku, kv > 0$.

$$\arg(kw) = \tan^{-1} \frac{kv}{ku} = \tan^{-1} \frac{v}{u} = \arg(w) \quad \square$$

(a)(ii) $OR \perp OQ$ (rotation by 90° anticlockwise), $|OR| = |OQ| = k|OP| > |OP|$.



(b)(i)

$$\begin{aligned} f(f(z)) - f(z) &= (z^2 - 2z)^2 - 2(z^2 - 2z) - (z^2 - 2z) \\ &= (z^2 - 2z)(z^2 - 2z - 3) \\ &= z(z - 2)(z - 3)(z + 1) \quad \square \end{aligned}$$

(b)(ii) Rotating \overrightarrow{BC} anticlockwise by $\frac{\pi}{2}$ about B gives $m\overrightarrow{BA}$:

$$\frac{f(f(z)) - f(z)}{f(z) - z} = \frac{z(z - 2)(z - 3)(z + 1)}{z(z - 3)} = (z - 2)(z + 1) = mi \quad \square$$

(b)(iii) With $z = x + 2i$:

$$(x^2 - x - 6) + i(4x - 2) = mi$$

Real part: $x^2 - x - 6 = 0 \implies (x - 3)(x + 2) = 0 \implies x = 3$ (since $x > 0$).

Imaginary part: $m = 4(3) - 2 = 10$.

$$B = f(3 + 2i) = (3 + 2i)^2 - 2(3 + 2i) = 9 + 12i - 4 - 6 - 4i = -1 + 8i$$

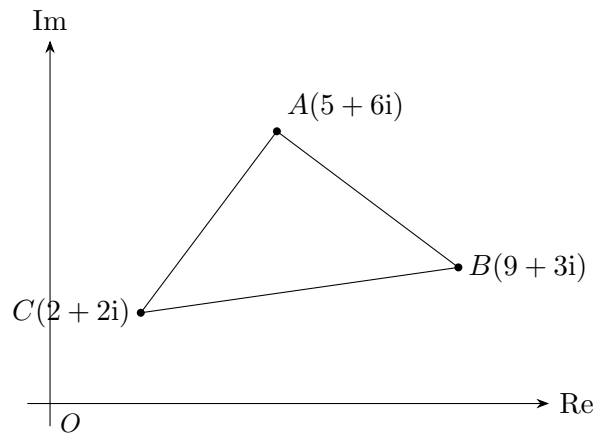
$x = 3, m = 10; \quad B$ represents $-1 + 8i$

Q15 RVHS 2025 Paper 2 Q2

••• [5 marks]

(a) AC is AB rotated 90° clockwise about A :

$$z_C - z_A = -i(z_B - z_A) \implies z_C = (5 + 6i) - i(4 - 3i) = 2 + 2i$$



(b)

Method 1 midpoint Midpoint of AD = midpoint of BC : $z_D = z_B + z_C - z_A = (9 + 3i) + (2 + 2i) - (5 + 6i) = 6 - i$

Method 2 vector $\overrightarrow{BD} = \overrightarrow{AC}$: $z_D = z_B + z_C - z_A = 6 - i$

$$z_C = 2 + 2i; \quad z_D = 6 - i$$

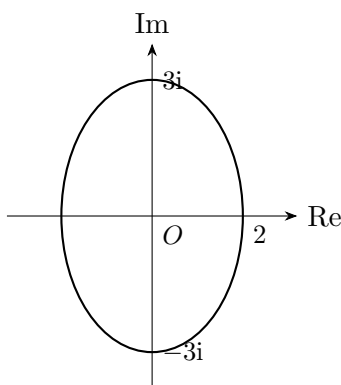
Section D Modulus-Argument & Loci

Q16-Q20

Q16 DHS 2025 Paper 1 Q8

••• [11 marks]

(a) $\cos t = x/2$, $\sin t = y/3$, so $\frac{x^2}{4} + \frac{y^2}{9} = 1$ (ellipse, semi-axes 2 and 3).



(b) From the diagram, three possible positions for z_2 :

- z_2 in 1st quadrant symmetric about Im-axis: $\arg(z_1 + z_2) = \frac{\pi}{2}$
- $z_2 = -z_1$: $z_1 + z_2 = 0$, arg undefined
- z_2 in 4th quadrant symmetric about Re-axis: $\arg(z_1 + z_2) = 0$

$$\arg(z_1 + z_2) \in \left\{ 0, \frac{\pi}{2}, \text{undefined} \right\}$$

(c) $\alpha = -(z_1 + z_2)$. If z_2 is at position 1, α is not real. Hence it is **not necessary** for α to be real.

(d) Since α not real, $z_2 = (-a, b)$ where $z_1 = (a, b)$. Using $a^2 + b^2 = \frac{26}{4}$ and the ellipse equation:

$$\frac{a^2}{4} + \frac{b^2}{9} = 1 \quad \text{with} \quad a^2 = \frac{26}{4} - b^2 \implies b^2 = \frac{9}{2}, \quad a^2 = 2$$

$$\alpha = -2bi = -2\sqrt{\frac{9}{2}}i = -3\sqrt{2}i \quad \beta = -(a^2 + b^2) = -\frac{13}{2}$$

$$\alpha = -3\sqrt{2}i; \quad \beta = -\frac{13}{2}$$

Q17 NYJC 2025 Paper 1 Q6

••• [8 marks]

(a) If $|p| = |q|$, $OPRQ$ forms a **rhombus** (two equal-length sides from O). The diagonal OR bisects $\angle POQ$:

$$\arg(r) = \frac{1}{2}(\alpha + \beta)$$

(b) Q' is the reflection of Q in line OP , so $\angle POQ' = \angle POQ = \beta - \alpha$.

(b)(i)

$$\arg(q') = \alpha - (\beta - \alpha) = 2\alpha - \beta$$

(b)(ii) $|q'| = |q|$. With $\arg(q') = 2\alpha - \beta$:

$$q' = |q| \cos(2\alpha - \beta) + i|q| \sin(2\alpha - \beta)$$

Common error / Examiner note: $\beta > 2\alpha$ means $2\alpha - \beta < 0$. The argument of q' is negative (below real axis). Do not write $\arg(q') = \beta - 2\alpha$ with the sign reversed the vector has been reflected below the real axis.

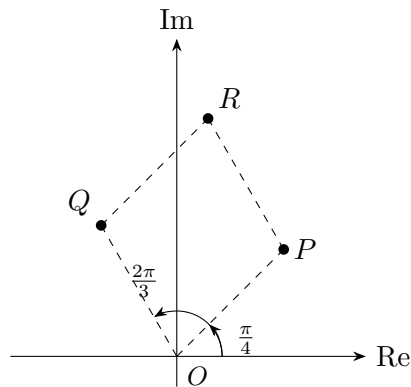
Q18 ASRJC 2025 Paper 2 Q6

••• [10 marks]

(a)

$$|p| = \sqrt{(\sqrt{2})^2 + (\sqrt{2})^2} = 2 \quad \arg(p) = \tan^{-1}\left(\frac{\sqrt{2}}{\sqrt{2}}\right) = \frac{\pi}{4}$$

$$|p| = 2, \quad \arg(p) = \frac{\pi}{4}$$

(b) Argand diagram ($|q| = 2$, $\arg(q) = \frac{2\pi}{3}$):

(c)

$$q = 2\left(\cos \frac{2\pi}{3} + i \sin \frac{2\pi}{3}\right) = -1 + \sqrt{3}i$$

$$\operatorname{Re}(q) = -1, \quad \operatorname{Im}(q) = \sqrt{3}$$

(d) Since $|p| = |q| = 2$, $OPRQ$ is a **rhombus**.(e) By the rhombus property, $\arg(p+q) = \frac{\pi/4 + 2\pi/3}{2} = \frac{11\pi}{24}$.Also $p+q = (\sqrt{2}-1) + (\sqrt{2}+\sqrt{3})i$, so:

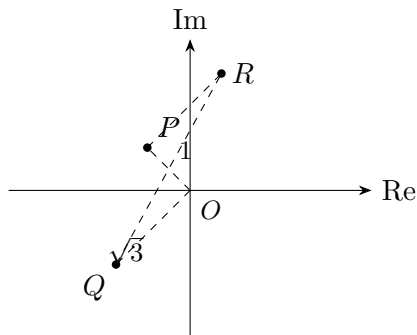
$$\tan \frac{11\pi}{24} = \frac{\sqrt{2} + \sqrt{3}}{\sqrt{2} - 1} = \frac{(\sqrt{2} + \sqrt{3})(\sqrt{2} + 1)}{(\sqrt{2})^2 - 1^2} = \sqrt{6} + \sqrt{3} + \sqrt{2} + 2 \quad \square$$

Q19 VJC 2025 Paper 2 Q4

●●● [9 marks]

(a) $w = i\sqrt{3}z$: $|w| = \sqrt{3}|z| = \sqrt{3}$, $\arg(w) = \theta + \frac{\pi}{2}$.

$OQ \perp OP$, $|OQ| = \sqrt{3}|OP|$. $ORPQ$ is a parallelogram (rectangle).



$ORPQ$ is a rectangle; Area of $ORPQ = 1 \times \sqrt{3} = \sqrt{3}$

(c) $z = \cos \frac{3\pi}{4} + i \sin \frac{3\pi}{4}$:

$$z = -\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i$$

(d)

$$\begin{aligned} z - w &= z(1 - i\sqrt{3}) = \left(-\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i\right)(1 - \sqrt{3}i) \\ &= \frac{1}{\sqrt{2}}(-1 + i)(1 - \sqrt{3}i) = \frac{1}{\sqrt{2}}[(\sqrt{3} - 1) + (\sqrt{3} + 1)i] \end{aligned}$$

$$k = \frac{1}{\sqrt{2}}$$

(e) Since $z - w$ lies in the 1st quadrant and $\arg(z - w) = \frac{3\pi}{4} - \frac{\pi}{3} = \frac{5\pi}{12}$:

$$\tan \frac{5\pi}{12} = \frac{\frac{1}{\sqrt{2}}(\sqrt{3} + 1)}{\frac{1}{\sqrt{2}}(\sqrt{3} - 1)} = \frac{\sqrt{3} + 1}{\sqrt{3} - 1} \quad \square$$

Q20 NJC 2025 Paper 2 Q1

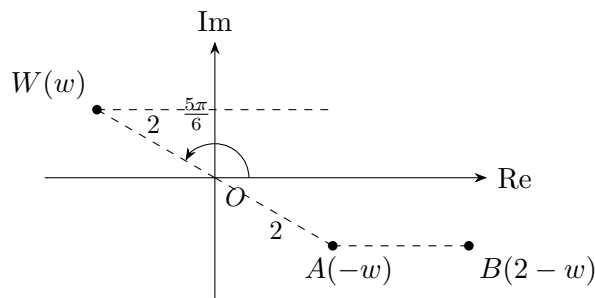
••• [7 marks]

(i) $w = -\sqrt{3} + i$, so:

$$|w| = \sqrt{3+1} = 2 \quad \arg(w) = \pi - \tan^{-1} \frac{1}{\sqrt{3}} = \pi - \frac{\pi}{6} = \frac{5\pi}{6}$$

$$|w| = 2, \quad \arg(w) = \frac{5\pi}{6}$$

(ii)



W , O and A are collinear (through origin). AB is parallel to the real axis. Triangle OAB is isosceles with $|OA| = |OB| = 2$.

(iii) Let $\angle AOB = \angle ABO = \theta$ (base angles of isosceles triangle).

$$2\theta + \left(\pi - \frac{5\pi}{6}\right) = \pi \implies 2\theta = \frac{5\pi}{6} - \frac{\pi}{6} \cdot 0 \implies 2\theta = \frac{\pi}{6} \implies \theta = \frac{\pi}{12}$$

$$\arg(2-w) = -\frac{\pi}{12}$$