A Maths Formula Sheet: G3 Additional Mathematics (K341)

Shaded: given in the exam. Everything else: memorise.

Given: 1. Algebra

- $ax^2+bx+c=0 \Rightarrow x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$
- $(a+b)^n=a^n+\binom{n}{1}a^{n-1}b+\binom{n}{2}a^{n-2}b^2+\cdots+\binom{n}{r}a^{n-r}b^r+\cdots+b^n$, n a positive integer
- $\binom{n}{r}=\frac{n!}{r!\,(n-r)!}=\frac{n(n-1)\ldots(n-r+1)}{r!}$

Given: 2. Trigonometry

- $\sin^2 A+\cos^2 A=1,\ \sec^2 A=1+\tan^2 A,\ \operatorname{cosec}^2 A=1+\cot^2 A$
- $\sin(A\pm B)=\sin A\cos B\pm\cos A\sin B$
- $\cos(A\pm B)=\cos A\cos B\mp\sin A\sin B$
- $\tan(A\pm B)=\frac{\tan A\pm\tan B}{1\mp\tan A\tan B}$
- $\sin 2A=2\sin A\cos A,\ \tan 2A=\frac{2\tan A}{1-\tan^2 A}$
- $\cos 2A=\cos^2 A-\sin^2 A=2\cos^2 A-1=1-2\sin^2 A$
- $\triangle ABC$: $\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C},\ a^2=b^2+c^2-2bc\cos A,\ \Delta=\frac{1}{2}bc\sin A$

1. Simultaneous equations

- Make x or y the subject of the linear equation; substitute into the other

2. Indices and surds

- $a^m a^n=a^{m+n},\ a^m\div a^n=a^{m-n},\ (a^m)^n=a^{mn},\ (ab)^n=a^n b^n$
- $a^0=1,\ a^{-n}=\frac{1}{a^n},\ a^{\frac{m}{n}}=(\sqrt[n]{a})^m$
- $\sqrt{a}\sqrt{b}=\sqrt{ab},\ \frac{\sqrt{a}}{\sqrt{b}}=\sqrt{\frac{a}{b}},\ (\sqrt{a}+\sqrt{b})(\sqrt{a}-\sqrt{b})=a-b$
- Surd equations: square, then check every answer

3. Polynomials

- $\mathrm{f}(x)=\text{divisor}\times\text{quotient}+\text{remainder}$
- Remainder of $\mathrm{f}(x)\div(ax-b)$ is $\mathrm{f}(\frac{b}{a})$; factor if $\mathrm{f}(\frac{b}{a})=0$
- $a^3\pm b^3=(a\pm b)(a^2\mp ab+b^2)$

4. Partial fractions

- $\frac{px+q}{(ax+b)(cx+d)}=\frac{A}{ax+b}+\frac{B}{cx+d}$
- $\frac{px^2+qx+r}{(ax+b)(cx+d)^2}=\frac{A}{ax+b}+\frac{B}{cx+d}+\frac{C}{(cx+d)^2}$
- $\frac{px^2+qx+r}{(ax+b)(x^2+c^2)}=\frac{A}{ax+b}+\frac{Bx+C}{x^2+c^2}$
- Improper fraction: divide first

5. Quadratic equations and inequalities

- $b^2-4ac>0$ two real distinct roots; $=0$ equal roots; <0 no real roots; real roots: $\geqslant 0$
- Line and curve: >0 two points, $=0$ tangent, <0 no intersection
- Always positive: $a>0$, $b^2-4ac<0$; always negative: $a<0$, $b^2-4ac<0$
- $ax^2+bx+c=a(x+\frac{b}{2a})^2+c-\frac{b^2}{4a}$
- $\alpha<\beta$: $(x-\alpha)(x-\beta)<0\iff\alpha<x<\beta$; $>0\iff x<\alpha$ or $x>\beta$

6. Exponential and logarithmic functions

- $y=a^x\iff x=\log_a y;\ \log_a 1=0,\ \log_a a=1$
- $\log_a xy=\log_a x+\log_a y,\ \log_a\frac{x}{y}=\log_a x-\log_a y,\ \log_a x^n=n\log_a x$
- $\log_a b=\frac{\log_c b}{\log_c a},\ \log_a b=\frac{1}{\log_b a};\ \mathrm{e}^{\ln x}=x,\ \ln\mathrm{e}^x=x$

7. Coordinate geometry

- $m=\frac{y_2-y_1}{x_2-x_1}$, length $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$, midpoint $(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2})$
- Parallel $m_1=m_2$; perpendicular $m_1m_2=-1$; $y-y_1=m(x-x_1)$
- Area $=\frac{1}{2}|(x_1y_2+\cdots+x_ny_1)-(y_1x_2+\cdots+y_nx_1)|$

8. Linear law

- $y=ax^n$: $\lg y=n\lg x+\lg a$; $y=kb^x$: $\lg y=(\lg b)x+\lg k$

9. Circles

- $(x-a)^2+(y-b)^2=r^2$: centre (a,b), radius r
- $x^2+y^2+2gx+2fy+c=0$: centre $(-g,-f)$, radius $\sqrt{g^2+f^2-c}$
- Tangent $\perp$ radius; perpendicular bisector of a chord passes through the centre

10. Binomial expansions

- $(r+1)$th term $\binom{n}{r}a^{n-r}b^r$; term independent of x: power of x is 0

11. Trigonometry

- $\tan A=\frac{\sin A}{\cos A}$, $\cot A=\frac{\cos A}{\sin A}$, $\sec A=\frac{1}{\cos A}$, $\operatorname{cosec}A=\frac{1}{\sin A}$
- $\sin 30^\circ=\frac{1}{2}$, $\cos 30^\circ=\frac{\sqrt{3}}{2}$, $\tan 30^\circ=\frac{1}{\sqrt{3}}$; $\sin 45^\circ=\cos 45^\circ=\frac{\sqrt{2}}{2}$, $\tan 45^\circ=1$
- $\sin 60^\circ=\frac{\sqrt{3}}{2}$, $\cos 60^\circ=\frac{1}{2}$, $\tan 60^\circ=\sqrt{3}$; $\pi\text{ rad}=180^\circ$
- Quadrants: all, sin, tan, cos positive; angles $180^\circ-\alpha$, $180^\circ+\alpha$, $360^\circ-\alpha$
- $\sin(-A)=-\sin A$, $\cos(-A)=\cos A$, $\tan(-A)=-\tan A$, $\sin(90^\circ-A)=\cos A$
- $\sin^{-1}x\in[-90^\circ,90^\circ]$, $\cos^{-1}x\in[0^\circ,180^\circ]$, $\tan^{-1}x\in(-90^\circ,90^\circ)$
- $y=a\sin bx+c$: amplitude $|a|$, period $\frac{360^\circ}{b}$; $y=a\tan bx$: period $\frac{180^\circ}{b}$
- $a\cos\theta+b\sin\theta=R\cos(\theta-\alpha)$, $R=\sqrt{a^2+b^2}$, $\tan\alpha=\frac{b}{a}$
- $\sin^2 A=\frac{1-\cos 2A}{2}$, $\cos^2 A=\frac{1+\cos 2A}{2}$

12. Differentiation

- $\frac{\mathrm{d}}{\mathrm{d}x}x^n=nx^{n-1}$; $\sin x\to\cos x$, $\cos x\to-\sin x$, $\tan x\to\sec^2 x$
- $\mathrm{e}^x\to\mathrm{e}^x$, $\ln x\to\frac{1}{x}$; $\frac{\mathrm{d}}{\mathrm{d}x}\mathrm{f}(ax+b)=a\,\mathrm{f}'(ax+b)$
- $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{\mathrm{d}y}{\mathrm{d}u}\cdot\frac{\mathrm{d}u}{\mathrm{d}x}$, $(uv)'=uv'+vu'$, $\left(\frac{u}{v}\right)'=\frac{vu'-uv'}{v^2}$
- Tangent $y-y_1=m(x-x_1)$; normal gradient $-\frac{1}{m}$; increasing $\frac{\mathrm{d}y}{\mathrm{d}x}>0$
- Stationary $\frac{\mathrm{d}y}{\mathrm{d}x}=0$: $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}>0$ min, <0 max; $\frac{\mathrm{d}y}{\mathrm{d}t}=\frac{\mathrm{d}y}{\mathrm{d}x}\cdot\frac{\mathrm{d}x}{\mathrm{d}t}$

13. Integration and kinematics

- $\int x^n\,\mathrm{d}x=\frac{x^{n+1}}{n+1}+c$, $\int(ax+b)^n\,\mathrm{d}x=\frac{(ax+b)^{n+1}}{a(n+1)}+c$, $n\neq-1$
- $\int\frac{1}{x}\,\mathrm{d}x=\ln x+c$, $\int\frac{1}{ax+b}\,\mathrm{d}x=\frac{1}{a}\ln(ax+b)+c$
- $\int\sin x\,\mathrm{d}x=-\cos x+c$, $\int\cos x\,\mathrm{d}x=\sin x+c$, $\int\sec^2 x\,\mathrm{d}x=\tan x+c$
- $\int\sin(ax+b)\,\mathrm{d}x=-\frac{1}{a}\cos(ax+b)+c$, $\int\cos(ax+b)\,\mathrm{d}x=\frac{1}{a}\sin(ax+b)+c$
- $\int\mathrm{e}^{ax+b}\,\mathrm{d}x=\frac{1}{a}\mathrm{e}^{ax+b}+c$; $\int_a^b\mathrm{f}(x)\,\mathrm{d}x=\mathrm{F}(b)-\mathrm{F}(a)$
- Area $\int_a^b y\,\mathrm{d}x$ (take the size below the x-axis); between curve and line $\int_a^b(y_{\text{upper}}-y_{\text{lower}})\,\mathrm{d}x$
- $v=\frac{\mathrm{d}s}{\mathrm{d}t}$, $a=\frac{\mathrm{d}v}{\mathrm{d}t}$; $s=\int v\,\mathrm{d}t$, $v=\int a\,\mathrm{d}t$; rest: $v=0$; distance: split at $v=0$

14. Proofs in plane geometry

- Tangent-chord (alternate segment) theorem; midpoint theorem
- Angle at centre $=2\times$ angle at circumference; same segment; semicircle 90°; cyclic quadrilateral 180°
- Tangent $\perp$ radius; equal tangents; perpendicular from centre bisects chord
- Congruent: SSS, SAS, AAS, RHS; similar: AA, three sides in ratio, SAS in ratio